
Development of Empirical Yield Curves For the PJ1, SP1, SB1, PO1, and SF1 Standard Forest Units

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Abstract

The interim results of the Benchmark Yield Curve Project are presented. In order to provide credible yield curves, all available growth and yield data were used. Equations were fit to the data to predict the growth and yield of the standard forest units. The equations use forest resource inventory data to predict forest unit development.

Basal area and stems numbers were predicted by standard forest unit as a function of age, site index, and stocking. Top height was predicted using published site index curves. Gross total volume was estimated as a function of stand basal area and top height while gross merchantable volume was estimated as a function of gross total volume and age.

Basal area growth was estimated as a function of current basal area and site index. The rate of change in stem numbers was estimated as a function of age and basal area.

Mixed species forest units are generally not dominated by a single species and a procedure is recommended for developing yield curves for the mixed species forest units.

The results are compared to existing curves in the context of the Romeo Malette Forest.

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Most modeling efforts commence with any data available, and the modeling approach often may be dictated by limitations of the data. Many models owe much of their success to the foresight and dedication of our forebears who established permanent plots and maintained both plots and measurement records carefully. (Vanclay 1994, p79).

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1 Objective

The purpose of this project is to develop yield curves based on field data for the analysis of wood supply. These curves should represent a range of current and potential management intensities (Table 1) for the standard northeast forest units (Watt et al. 2001) and southcentral (draft) forest units. The curves should be compatible with the sustainable forest management model (SFMM) (Davis 1999) and be applicable to forest management units (FMUs) within Tembec's operating area in Ontario.

The intent of the project was to use all available growth and yield plot data.

2 Background

The Ontario Forest Accord (OMNR 1999) outlined 31 commitments agreed to by members of the forest industry, the Partnership for Public Lands, and the Ontario Ministry of Natural Resources. Commitment 5 commits to the development of an Ontario forest science partnership, in part to assess the impacts of intensive forest management on increased forest growth and yield. Tembec's response to this commitment led to the creation of the Forestry Research Partnership (FRP) that includes the Canadian Forest Service, the Ontario Ministry of Natural Resources and Tembec.

In an Intensive Forest Management (IFM) workshop in Sault Ste. Marie in 1999, participants noted the following needs (Bell et al. 2000, p31).

- Locally calibrated (region-, subregion-, and FMU-specific) yield curves for the full spectrum of silvicultural treatment options. These curves should be based on forest units and ecosites.
- Yield curves for mixedwood stands.
- Yield curves for managed stands including plantations established with improved stock and/or effective vegetation management.
- Yield curves for partial harvesting and thinning regimes.

Participants also identified the need for objective, peer-reviewed programs and projects in order to gain reliable information.

2.1 *Intensive forest management options (NEBIEs)*

Bell et al. (2000) summarized definitions of levels of intensity of forest management and silviculture, drawing largely from Natural Resources Canada (1995) and Dunster and Dunster (1996). In addition to the extensive, basic and intensive levels described in Bell et al. (2000), natural (no intervention) and elite (beyond intensive and possibly still experimental) intensities are of interest.

For the purposes of this project, the management intensities for the various forest units have been put in a standard framework with Natural, Extensive, Basic, Intensive, and Elite (NEBIE) levels (Table 1). These management intensities focus on silvicultural manipulations to achieve a target. Site modification (e.g., drainage, fertilization) and tree improvement are two additional dimensions of management intensity. Site modification

is not considered here. Tree improvement gains are available for jack pine and black spruce and are included as simple height gains in the “intensive” management level.

Table 1. The standard framework for the management intensities is given.

Level	Target/Focus	Planned Activity
Natural	Same conditions as natural	Present forest condition
Extensive	Manipulating species composition.	Natural regeneration (may include assisted natural regeneration (scarification and cone scattering))
Basic	Manipulating species composition Achieving full site occupancy.	Assisted natural regeneration and artificial regeneration (may include site prep and vegetation management) (e.g. SIP and planting with no other planned intervention after FTG)
Intensive	Manipulating species composition Achieving full site occupancy Controlling density to optimize individual tree growing space	Assisted natural regeneration and artificial regeneration featuring density regulation at a young age (may include site prep and vegetation management) (e.g. SIP and planting with planned spacing/thinning intervention after FTG stage)
Elite	Achieving higher product value	Practices designed to enhance the value or quality of the forest product (e.g. pruning, crop tree management)

Some management actions will result in a stand moving from one forest unit to another. In some cases, a low level of silvicultural investment will result in a conversion from a conifer unit to a mixedwood unit. In other cases, a higher level of silvicultural investment will result in the rehabilitation of a mixedwood unit to a conifer unit. Some forest units will secede to another forest unit with age. This project focuses on stand development within a forest unit. If a stand moves to another forest unit following harvest, the yield curve for the new forest unit should be used to predict development.

Management intensities for some northeast forest units are given in Table 2.

Table 2. Management intensities are given for the PJ1, SB1, SP1, SF1, and PO1 forest units. The forest unit descriptions are taken from Watt et al. (2001) while the species composition, stocking, and site class for the natural conditions are taken from the Romeo Malette Forest. The remaining intensities are hypothetical but within the current range of practise.

FU	Intensity	Target ¹	Silviculture
PJ1	Natural	Average of the forest unit Spp Pj ₈₂ Sb ₉ Po ₆ Bw ₄ , stocking = 0.89, SC = 1.7	Natural (unknown) origin
	Extensive	70% or greater jack pine, poorer stocking Spp Pj ₈₂ Sb ₉ Po ₆ Bw ₄ , stocking 90% of natural, SC = 1.7	Cut-origin
	Basic	Unmanaged Pj plantation Spp Pj ₈₂ Sb ₉ Po ₆ Bw ₄ , SC = 1.7	Scarification and plant 2000 stems/ha
	Intensive 1	Pine plantation with well-spaced trees	Plant Pj 2000 stems/ha, PCT every third row at age 20, CT 1/3 of BA at age 35
	Intensive 2	Same as intensive 1 + improved stock	3% ² first generation height gain

¹ The species composition is given as a percentage, the stocking is the basal area relative to a normal forest as defined in Plonski (1981) and SC is site class, also defined in Plonski (1981).

FU	Intensity	Target ¹	Silviculture
	Intensive 3 Elite	Same as intensive 1 + improved stock Increased value	8% gain from clonal orchard Pruning?
SP1	<i>Upland black spruce dominated conifer stands on fresh to moist, medium loamy to clayey soils.</i>		
	Natural	Average of the forest unit Spp Sb ₆₀ Bw ₁₄ Po ₉ Pj ₈ Bf ₆ Ce ₂ Sw ₂ , stocking = 0.65, SC = 1.1	Natural (unknown) origin
	Extensive	Similar to natural, poorer stocking Spp Sb ₆₀ Bw ₁₄ Po ₉ Pj ₈ Bf ₆ Ce ₂ Sw ₂ , stocking = 0.55, SC = 1.1	Cut-origin
	Basic	Well stocked Spp Sb ₆₀ Bw ₁₄ Po ₉ Pj ₈ Bf ₆ Ce ₂ Sw ₂ , stocking = 0.75, SC = 1.1	Scarification
	Intensive Sw	Sw plantation with well-spaced trees Spp Sw ₈₀ Pj ₁₀ Po ₁₀	Plant Sw 1800 stems/ha, CT 1/3 of BA at age 35
	Intensive 1 Sb	Sb plantation with well-spaced trees Spp Sb ₈₀ Pj ₁₀ Po ₁₀	Plant Sb 1800 stems/ha, CT 1/3 of BA at age 35
	Intensive 2	Same as intensive 1 Sb + improved stock	9% first generation height gain
	Intensive 3 Elite	Same as intensive 1 + improved stock Increased value	15% gain from clonal orchard Pruning?
SB1	<i>Nearly pure black spruce growing on wet deep organic soils and on moist peaty-phased mineral soils in lower slope positions.</i>		
	Natural	Average of the forest unit Spp Sb ₉₁ Ce ₂ La ₂ Po ₂ Bw ₂ , stocking = 0.59, SC = 1.4	Natural (unknown) origin
	Extensive	70% or greater jack pine, poorer stocking Spp Sb ₉₁ Ce ₂ La ₂ Po ₂ Bw ₂ , stocking 0.49 of natural, SC = 1.4	Cut-origin
	Basic	Well stocked Spp Sb ₉₁ Ce ₂ La ₂ Po ₂ Bw ₂ , stocking 0.69 of natural, SC = 1.4	CLAAG, HARP, fill planting, planting skid trails, seeding
	Intensive	Well spaced, natural origin Spp Sb ₉₁ Ce ₂ La ₂ Po ₂ Bw ₂ , stocking 0.69 of natural, SC = 1.4	Commercial thin 1/3 of BA at age 35
	Elite	Increased value	?????
PO1	<i>Hardwood stands dominated by trembling aspen. They typically occur on fresh to moist loamy to clayey soils with free carbonates present in the upper 100cm.</i>		
	Natural	Average of the forest unit Spp Po ₇₀ Bw ₁₁ Sb ₈ Pj ₄ Sw ₃ Bf ₃ stocking=0.78, SC = 1.7	
	Extensive	Similar to Natural, poorer stocking, lower aspen Spp Po ₆₅ Bf ₁₃ Bw ₁₁ Sb ₈ Pj ₄ Sw ₃ stocking 0.68 of natural, SC = 1.7	Timing of harvest to penalize aspen regeneration
	Basic	Similar to Natural, improved stocking, higher aspen Spp Po ₈₀ Bw ₆ Sb ₅ Pj ₄ Sw ₃ Bf ₃ stocking 0.88 of natural, SC = 1.7	Timing of harvest to favour aspen regeneration
	Intensive	Same as basic with a commercial thinning at age 15 to remove 1/3 of the basal area	
	Elite	Hybrid poplar	
SF1	<i>Mixed conifer stands of white spruce, balsam fir, black spruce and eastern white cedar growing on moist sandy to clayey soils. They are often found on lower slope positions associated with telluric seepage.</i>		
	Natural	Average of the forest unit Spp Sb ₃₀ Sw ₁₈ Bf ₁₆ Bw ₁₅ Ce ₁₂ Po ₇ La ₁ stocking = 0.61, SC = 1.2	
	Extensive	Similar to Natural, poorer stocking Spp Sb ₃₀ Sw ₁₈ Bf ₁₆ Bw ₁₅ Ce ₁₂ Po ₇ La ₁ stocking 90% of natural, SC = 1.2	
	Basic	Similar to Natural, improved stocking	

² Unpublished data from the Ontario Ministry of Natural Resources. The genetic gains for jack pine height are less than the 7.5% and 5% gains reported in Newton (2003) for jack pine in Ontario. The genetic gains for black spruce height are less than the 9-18% reported in Newton (2003) for black spruce in Ontario.

FU	Intensity	Target ¹	Silviculture
		Spp Sb ₃₀ Sw ₁₈ Bf ₁₆ Bw ₁₅ Ce ₁₂ Po ₇ La ₁ stocking 110% of natural, SC = 1.2	
	Intensive	Same as basic with a thinning at age 35 to remove 1/3 of the basal area	
	Elite		

2.2 The forest

Yield curves are an integral part of forest management planning. This report presents preliminary results for the PJ1, SB1, SP1, PO1, and SF1 forest units. Although the definitions of the standard forest units have been agreed upon, the forest inventory attributes of a forest unit vary from management unit to management unit depending on the soil types, past disturbances and past management. Therefore, the yield curves will vary with management unit. The results are presented in the context of the Romeo Malette Forest (RMF) (Figure 1 and Table 3). The RMF is dominated by spruce and is licensed to Tembec whose main interest is in spruce and other softwoods. Most of the softwood is used for lumber and chips while the hardwood goes to the Malette oriented strand board plant in Timmins.



Figure 1. The location of the Romeo Malette Management Unit, the management unit used in this report for illustrating the results.

Table 3. A summary of the Romeo Malette Forest.

Species	Area (ha)	Age class (%)				AAC (m ³ /yr)	Product
		Regen	Immature	Mature	Overmature		
Spruce	265,988	29	28	30	13	232,886	Sawlogs/Pulp
Jack pine	48,570	39	11	47	3	66,563	Sawlogs/Pulp
Balsam fir	4,294	24	43	32	1	6,527	Sawlogs/Pulp
Poplar	65,221	25	18	52	5	123,908	Pulpwood
White birch	69,992	12	34	53	1	50,024	Pulpwood
Other conifer	22,376	8	11	44	37	Na	Na
Other Hardwood	668	0	44	40	16	Na	Na
Total	477,109					479,109	

2.3 The forest units

The RMF falls within the Northeast Planning Region so this project uses the standard forest units for the northeast region (Watt et al. 2001). In the forest management planning manual (OMNR 1995) a forest unit is defined as “an aggregation of forest stands for management purposes which have similar species composition, develop in a similar manner, (both naturally and in response to silvicultural treatments) and are managed under the same silvicultural system.”

2.4 Existing curves

The curves currently used in most forest management plans are derived from SFMMTool (Watkins and Davis 1999) which has three sets of yield curves – Plonski’s modified curves, Northeast Regional curves (by Neil Maurer), and custom or user-defined curves. Plonski’s (1981) curves are based on temporary sample plots with the bulk of the data collected prior to 1960. The tables were modified to project yields for older stands and to also reflect mortality following rotation age. Maurer’s curves were also based on temporary sample plots, primarily in natural, untreated stands.

It was anticipated that developing curves from repeated measurements on permanent sample plots and using all the plot data available in Ontario would provide improved yield estimates and, most importantly, provide empirical estimates of growth.

3 DATA

Plot locations are given in Figure 2 and the data are summarized in Table 4. The data come from the Ontario provincial database as well as the Canadian Forest Service and the Quebec Forest Service. Raw data were compiled to a standard form described elsewhere.

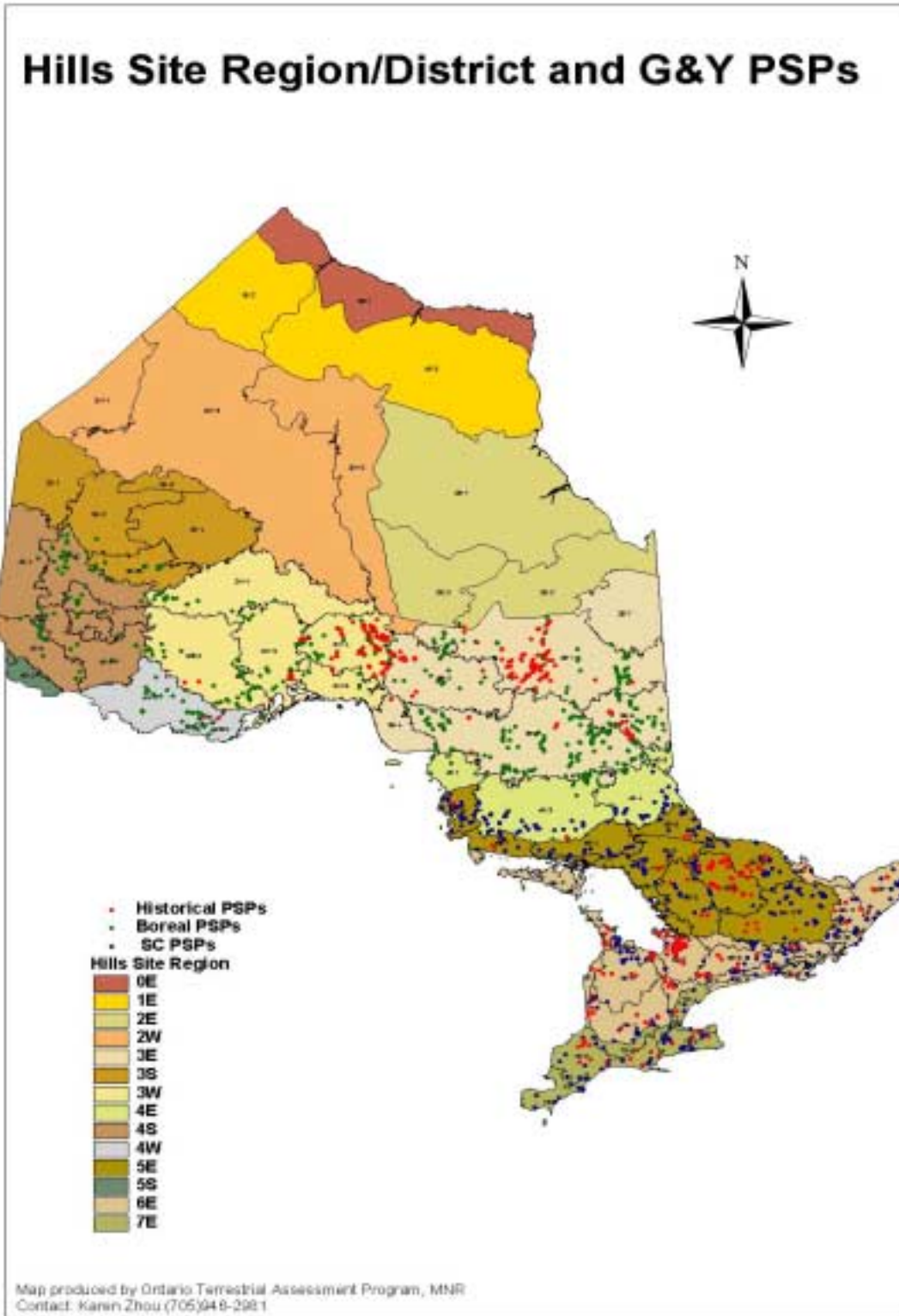


Figure 2. The plot locations are given by Hills site region. “SC” refers to southcentral.

3.1 Representativeness of the data

The data analysis focuses on ensuring the model forms fit the data. It is also essential that the data reflect the population. In general, historic plots like the AmericanCan and Kimberly Clark plots were located on better sites with high stocking. They tend to represent better than average conditions. Experimental sites such as the Thunder Bay spacing trial and those from the Petawawa Research Forest tend to have above average growth due to complete site occupancy, above average protection, and better tending. It is important that these attributes be included as covariates in the model to avoid bias. The more recently established growth and yield plots (1994+) are more representative of the range of sites in the population.

The Beckwith Robboelin plots at Limestone Lake were removed because they occur on limestone soils, not representative of Tembec's license areas, and have significantly higher growth.

The Quebec plots are a nested design. Small trees are measured on a small plot and the density (stems/ha) estimates had high variability. In general, the Quebec plots were not used to estimate the density functions.

Table 4. The plots are summarized by forest unit. Means are followed by the data ranges in brackets.

FU	Origin	Plots	Msmts ³	Age (years)	BA (m ² /ha)	Density (stems/ha)	Stocking ⁴	SI (m@age 50)	Volume (m ³ /ha)
PJ1	Natural	486 (1526)	980 (368)	58 (7-152)	18.3 (0-49.6)	1313 (25-6537)	0.74 (0 - 1.8)	15.7 (5-33.7)	136 (0-471.8)
	Planted	125	291 (11)	26 (6-59)	16.7 (0-35.2)	2272 (17- 6577)	0.76 (0 - 1.3)	20.1 (10.9-28.0)	107 (0-325)
SB1	Natural	1781 (1526)	3877 (2994)	96 (4-244)	17.3 (0-64.5)	1586 (2-14445)	0.52 (0-4.7)	9.9 (0-38.7)	100 (0-409)
SP1	Natural	733 (488)	1391 (751)	72 (5-211)	22.5 (0 -71.8)	1953 (75-23752)	0.71 (0-6.68)	13.7 (0-36.0)	146 (0-415)
	Planted Sw	161 (6)	347 (8)	25 (6-48)	16.7 (0-50.8)	2473 (25-14575)	1.63 (0-4.47)	6.1 (2.1-16.5)	80 (0-323)
	Planted Sb	172 (3)	336 (3)	26 (8-48)	15.6 (0.1-49.7)	3073 (100-10975)	1.3 (0.1-3.5)	14.0 (4.6-34.9)	69 (0-278)
PO1	Natural	760 (436)	1416 (729)	525 (2-186)	26.4 (0-68.9)	3289 (5-27775)	1.01 (0-3.95)	20.6 (10.0-35.0)	205 (0-650)
SF1	Natural	766 (734)	1159 (1114)	87 (5-216)	22.8 (0.3-55.7)	4464 (25-34100)	0.68 (0-5.76)	12.1 (0-37.3)	120 (1-335)

³ The total number of measurements is followed by in brackets by the number of measurements from Quebec.

⁴ See section 2.2 for an explanation of stocking.

3.2 Site index

The site index for jack pine was calculated using equation 22 of Carmean et al (2001) and top height was predicted using the equation 3 from the same source. For black and white spruce the site index curves were calculated from the equations in Appendix IV of Carmean (1996) and top height was predicted from Appendix III of the same source. Breast height age was calculated by subtracting 5 years from total age for plantations and 6 years from total age for natural stands. The poplar site index curve was taken from Carmean et al. (2002).

3.3 Stocking

For natural stands, the stocking for each plot was calculated as the ratio of the actual basal area compared to that predicted by Plonski (1981) for a fully stocked stand. The Plonski basal area was predicted using Payandeh's (1991) formulation of Plonski's yield tables. For jack pine, Payandeh's equation to predict the Plonski basal area is as follows.

$$(1) \quad \text{PlonskiBA} = 5.050 \cdot SI^{0.596} \cdot (1 - e^{-0.071})^{1.511}$$

There was a noticeable tendency for the average stocking to increase with age in the PJ1 and to decrease with age in the SP1 (Figure 3). This may have several causes:

- There may be a trend to normality. That is, lower density stands, on average, exhibit lower rates of mortality than higher density stands of the same age leading to a convergence over time if density related mortality dominates.
- Young stands may have a different definition of stocking. For example, the inventory stocking may come from silvicultural records and be based on stocked quadrats.
- In young stands, some trees may be below the lower dbh limit.
- Stand measurement protocols have changed. In general, the minimum dbh of tree measured has decreased. Some of the older measurements, in particular, may have had a minimum dbh of 10cm while the current standard is generally 2.5cm.
- In young stands, the Plonski basal area is rather small and smaller absolute differences in the actual basal area can lead to larger relative differences (stocking). For instance for PJ site class 3, a difference of 1m²/ha leads to a difference in stocking of 10% at age 20 but a difference of 4% at age 100.
- Historically, growth plots were sometimes abandoned if significant mortality occurs. The remaining growth plots tend to have higher than average basal areas and therefore stocking. This was partly compensated by a tendency toward harvesting the more productive plots.
- Stocking reductions with age are generally a result of tree mortality. For the SP1 there is a trend of decreasing site class with increase age so some of the older stands may be on less productive sites.

For plantations, stocking was calculated as the actual density divided by 2500 stems/ha.

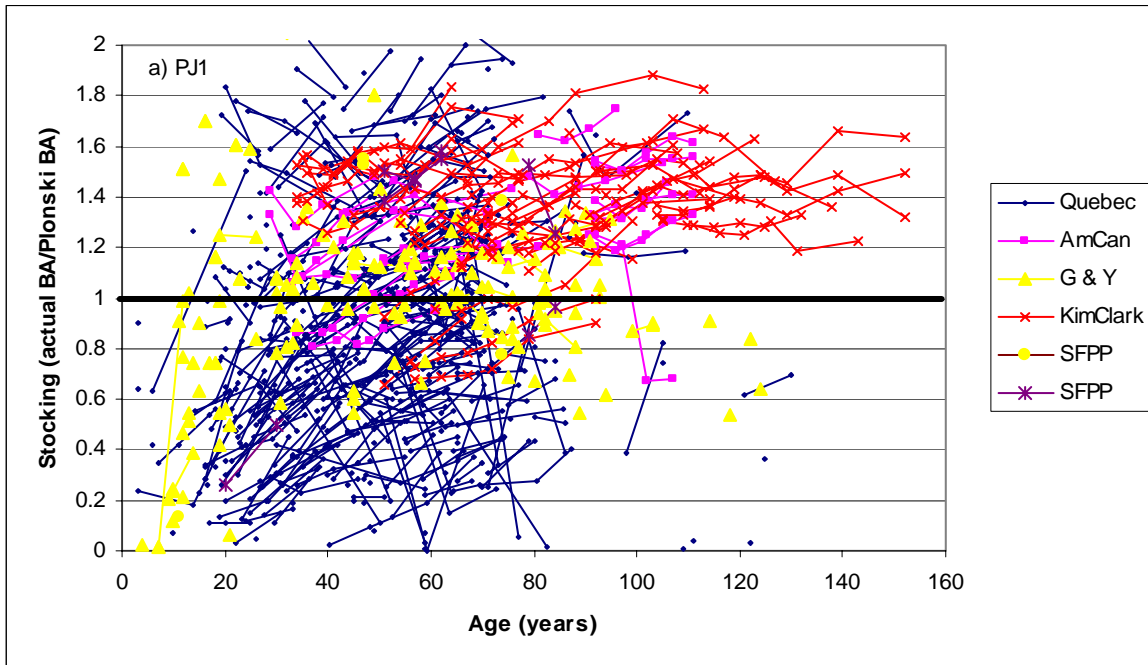
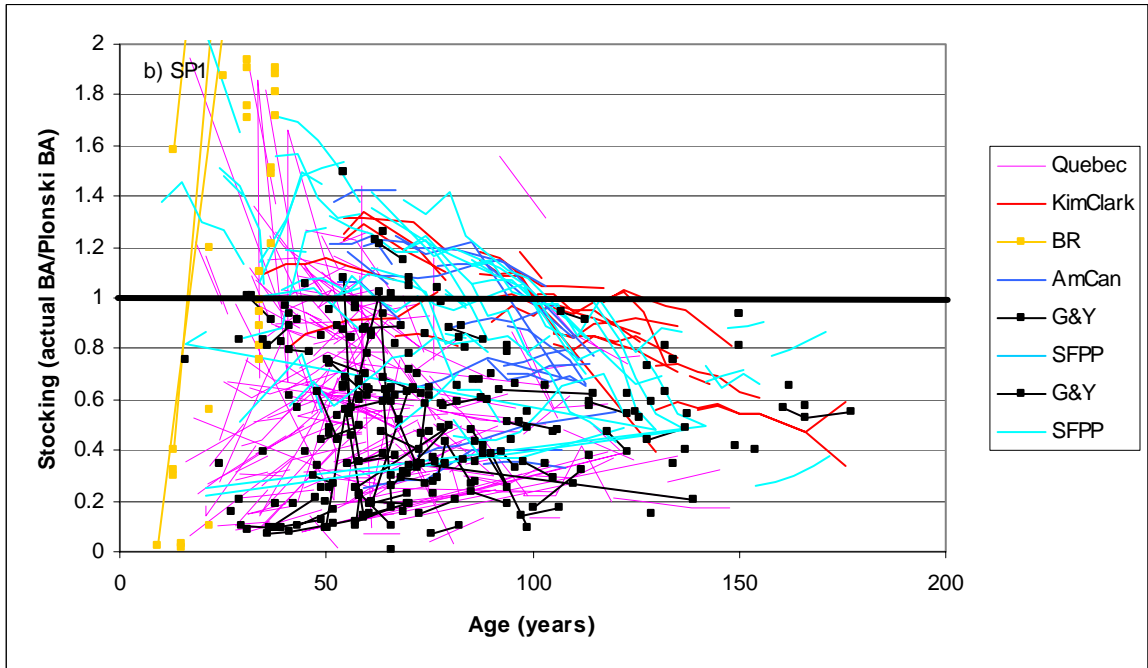


Figure 3. The trend of stocking with age is shown for the natural origin stands for the PJ1 forest unit (a) and for the SP1 forest unit (b).

Within the Romeo Malette Forest, there is a tendency for stocking to increase with age up to about 100 years and then start to decline (Figure 4). There are many other trends associated with overmature stands, including declines in site index and changes in species composition.

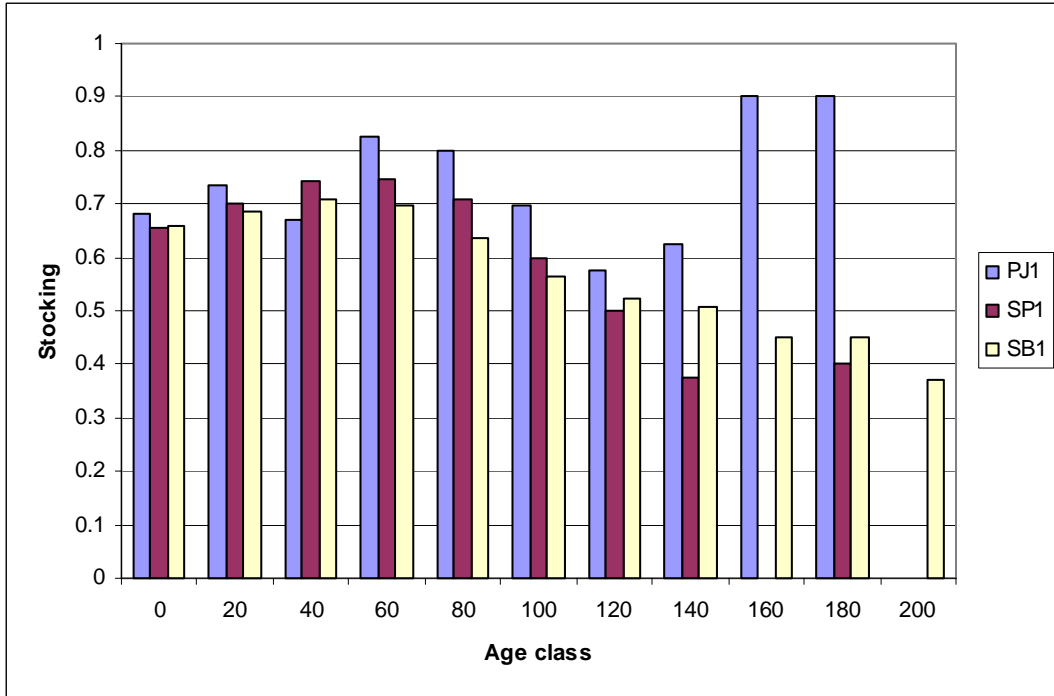


Figure 4. Average stocking is given by age class for the PJ1, SP1, and SB1 forest units in the Romeo Malette management unit. The data are taken from the forest resources inventory and the average is weighted by stand area. There is very little area in the PJ1 forest unit older than 120 years.

The following function was fit to the stocking-age data from the inventory to produce a smooth stocking curve for the yield curves. The observations were weighted by stand area.

$$(2) \quad \text{stocking} = \frac{b_2}{\sqrt{2\pi} \cdot b_1} \cdot e^{-\frac{(age-b_0)^2}{2 \cdot b_1^2}}$$

This is a scaled normal probability density function. A simple quadratic function also fit the data well but has the potential to predict negative stocking so equation (2) is preferred. Examples of the fit are given in Figure 5. Stands younger than 40 years and older than 160 years were not included, in accordance with the procedures in SFMMTool.

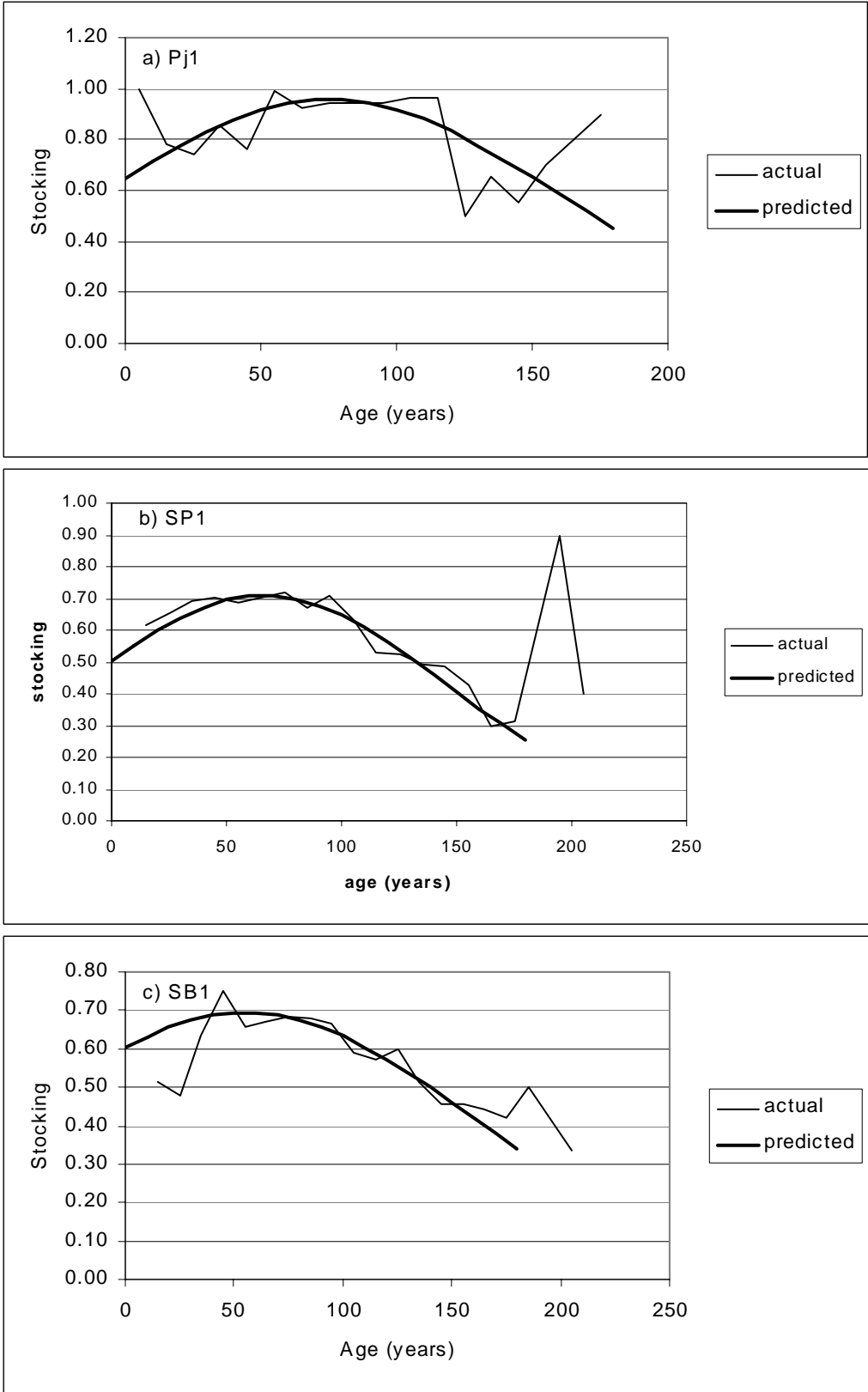


Figure 5. A comparison of the actual stocking and that predicted using equation (2).

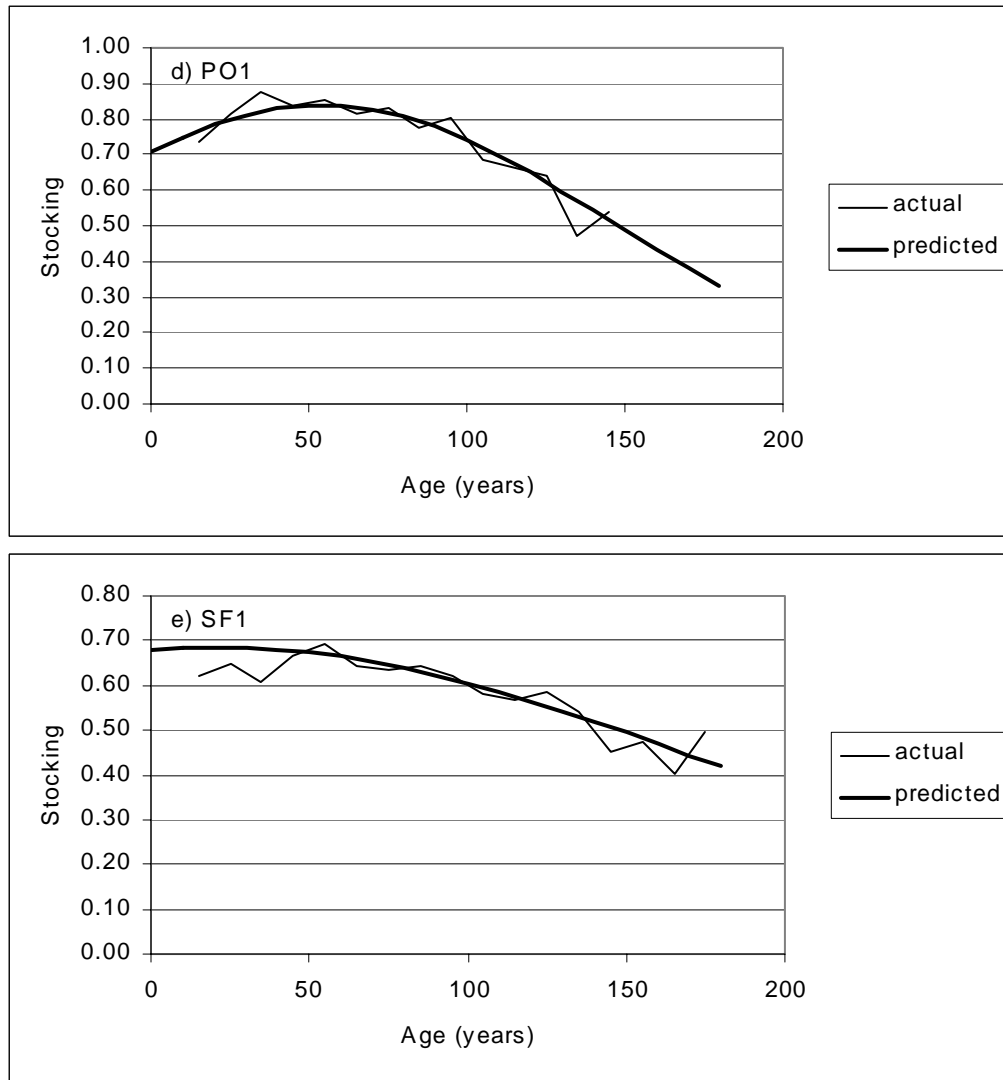


Figure 5 (cont.).

4 Methods

Yield curves must be compatible with forest resources inventory (FRI) data. The FRI data for each stand consist of the species composition (to the nearest 10%), the age class, stocking, average height of dominant and codominant trees, and site class. Various other classification variables are also available including ownership, site region, and site district. Generally, silvicultural history is not available as part of the FRI. The equations were fit by forest unit and origin (natural vs. planted stands).

Yield and growth were modeled. According to Vanclay (1994 p.18), yield equations assume a prescribed management regime. Growth equations have the advantage that silvicultural treatments such as thinning and spacing can be simulated at any time. The yield models should generally be appropriate for natural and extensive conditions with no management intervention. The yield curves may also be used to predict growth under

unmanaged conditions assuming that stocking is constant over time. The yield curves are based on all the data including plots with a single measurement.

The growth curves are recommended for prediction of the growth of managed stands. They are based on the following hypothesis: For a wide range of thinning regimes, the growth rates in a thinned stand are identical to those in an unthinned stand with the same characteristics (e.g., SI, basal area, top height). This is more likely to be the case for thinning from below or row thinnings where the residual stems have sufficient crown volume to respond relatively quickly to increased growing space. It may not be the case for stands that have long been over crowded, have small live crown ratios, and have increased susceptibility to windthrow following thinnings. As well, some stands may be at higher risk for damaging agents (e.g., insects and disease) or other negative influences (e.g., higher water table on wet, organic sites) following thinning. The expected gain from thinning is an increase in individual tree growth rates and a reduced time for reach a merchantable size.

The growth curves formulated and fit independently of the yield curves. There was no attempt ensure compatibility between the two prediction systems. The derivative of the yield equation could be used to predict growth but that assumes that a stand with certain values of the independent variables will maintain those values through the growth period. That may be acceptable for short prediction intervals but stocking shows a definite trend over longer time intervals.

Data compilation and analysis were conducted using SAS[®] BASE and STAT statistical software. The graphs were produced by putting the data and the equations into Microsoft Excel.

4.1 Selection of independent variables

The potential independent variables were age, site index, and stocking – attributes available from the inventory. Forest unit can be derived from the species composition in the inventory and stand origin should be available from silvicultural records so these attributes were also included.

4.2 Yield

4.2.1 Basal area

For even-aged forests, basal area increases with site index and stocking. Basal area also increases with age with a rapid increase at young ages that slows as the stand achieves full site occupancy. Basal area was predicted as a linear function of stocking and site class and a sigmoidal function of age using the following equation form.

$$(3) \quad \hat{BA} = \text{stocking} \cdot (\alpha_0 + \alpha_1 \cdot \hat{SI}) \cdot (1 - e^{-\beta_0 \cdot \text{age}^{\gamma_0}})$$

The $\text{stocking} \cdot (a_0 + a_1 \cdot SI)$ term represents the upper asymptote of basal area, the maximum basal area that a stand with that stocking and SI can achieve. The remainder of the equation predicts how rapidly the basal area approaches that maximum.

Equation (3) should be a relatively good predictor of basal area since the stocking is the ratio between actual and theoretical (Plonski) basal area. In the FRI the stocking is generally estimated from aerial photography, not calculated. The ramifications of this are not known. A study should be initiated to investigate potential links between FRI stocking and attributes measured on the sample plots.

4.2.2 Top height

Top height curves are an essential of part of the yield curves developed here. Historically, height data have been expensive to collect and were highly variable and generally deficient for minor species. Top height curves were taken from the literature and evaluated against the observed height development patterns in the data. Uncertainties and errors in the site index curves are not included in the yield prediction intervals.

Top height for jack pine was predicted using the following equation from Carmean (2001) equation 3.

$$(4) \quad \text{topht} = 1.3 + 4.1459 \cdot (\hat{SI} - 1.3)^{0.6224} \cdot (1 - K^{(age - age2bh)/50})^{1.3723 \cdot (\hat{SI} - 1.3)^{-0.0802}}$$

where

$$K = 1 - \left(\frac{\hat{SI} - 1.3}{4.1459 \cdot (\hat{SI} - 1.3)^{0.6224}} \right)^{\frac{1}{1.3723 \cdot (\hat{SI} - 1.3)^{-0.0802}}} \quad \text{and } age2bh = \text{age to breast height}$$

Top heights for the remaining species were predicted using equations from appendix III and IV of Carmean (1996). The equations associated with figure 11 of Carmean (1996) were used for black spruce. The equations associated with figure 17 of Carmean (1996) were used for white spruce (note that the white spruce plantation curves in Carmean (1996) were not used). The equations associated with figure 18 of Carmean (1996) were used for balsam fir. For aspen, the equations from Carmean et al. (2002) were used. Site index for aspen was determined using the same equation and a search algorithm. Site index was incremented in 0.5m units. The predictions from the equation for aspen in Carmean et al. (2002) are similar to those in Carmean (1996) up to approximately age 90 but then flatten out more quickly than Carmean (1996).

The age to reach breast height (*age2bh*) was assumed to be 6 years for natural stands and 5 years for plantations.

4.2.3 Density

Density changes with stocking, age, site index, and basal area. Several variations were tried including raising the independent variables to negative exponents and using the inverse of the independent variables. When comparing the alternative equations, particular attention was paid to the model behaviour at older ages.

The following equation form provided good predictions for density (stems/ha).

$$(5) \quad \hat{stems} = \frac{x_0}{SI} \cdot \text{stocking}^{-x_1} \cdot \hat{BA} \cdot \text{age}^{-x_2}$$

The variance of the residuals increased with predicted stems so the observations were weighted by $\frac{1}{\hat{BA}}$ resulting in a more homogenous variance for the residuals.

4.2.4 Volume

The following equation form was used to predict total stem volume (m³/ha). The coefficient is analogous to the cylindrical form factor $f = \frac{vol}{BA \cdot ht}$ (Husch et al. 1972).

$$(6) \quad \hat{vol} = z_0 \cdot \hat{BA} \cdot \text{topht}$$

The variance of the residuals increased with predicted volume so the observations were weighted by $\frac{1}{\hat{BA}}$ resulting in a more homogenous variance for the residuals.

Merchantable volumes were calculated using the scaling manual (OMNR 1995) minimum standards. That is, a stump height of 30 cm and a top diameter of 16cm (white & red pine, hemlock, poplar, or white birch), 10 cm (other conifer), or 20 cm (other hardwood). Initially merchantable volume was predicted as a simple proportion of the total volume. In order to capture the increase in the proportion of volume that is merchantable, an age term was added.

$$(7) \quad m\hat{vol} = z_3 \cdot (1 - e^{-z_1 \cdot \text{age}^{z_2}}) \cdot \hat{vol}$$

The relationship between net merchantable volume and age was obtained by fitting the following equation to the cull factors from OMNR (1978), by species. This predicts the cull fraction increases as a sigmoidal function of age.

$$(8) \quad \hat{cull}_i = (1 - e^{-d_0 \cdot \text{age}})^{d_1}$$

For jack pine, Table 9 from OMNR (1978) (for the central transition portion of the eastern ecological section) was used. For SB1, Table 5 of OMNR (1978) for lowland (wet) sites in the boreal forest region was used. For SP1, Table 4 of OMNR (1978) for upland sites was used except for white spruce (Intensive Sw) where Table 6 was used.

For each species, the relationship between the volume (GTV) to basal area ratio (VBAR), site index, and age was estimated using the entire data set.

For each species, the net merchantable volume was estimated using the following equation.

$$(9) \quad nm\hat{vol}_i = \frac{sppba_i \cdot \hat{cull}_i \cdot \hat{vbar}_i}{\sum_i sppba_i \cdot \hat{cull}_i \cdot \hat{vbar}_i} \cdot m\hat{vol} \quad \text{where } sppba_i \text{ is the basal area for species } i$$

The fraction of the net total volume in each of the products in Table 5 was estimated using the following equation.

$$(10) \quad \text{product_fraction} = (p_1 + p_2 \cdot \text{topht}) \cdot (1 - e^{-p_3 \cdot dbh_q^{p_4}}) \quad \text{where } dbh_q \text{ is quadratic mean dbh}$$

The SPF1 product predictions are conditional on the stand dbh_q which must be at least 16cm. For SPF2 product predictions, the dbh_q must be at least 12cm

Table 5. The following spruce/pine/fir (SPF) products were of interest for the PJ1, SP1, SF1, and SB1 forest units. All other species were considered pulp.

Product	Species	Small end diameter (cm)
SPF1	Pj, Sb, Sw, Bf, Sr, Sn	16+
SPF2	Pj, Sb, Sw, Bf, Sr, Sn	12-16
SPF3	Pj, Sb, Sw, Bf, Sr, Sn	10-12

4.3 Growth

Fewer data were available for estimating growth. The attributes at the start of a growth period were used to estimate the average annual growth during the growth period. The growth intervals were generally between 5 and 10 years. Most PSPs are on a 5 year remeasurement cycle. It is recommended that the equations are used to predict growth for a similar interval (5 to 10 years). The predictions in this report are based on a 5-year growth period.

4.3.1 Basal area

Basal area was predicted using the following equation, which is the differential form of the Chapman-Richards growth function (Pienaar and Turnbull 1973).

$$(11) \quad \frac{dBA}{dt} = \alpha \cdot BA^\beta - \gamma \cdot BA$$

This model was proposed by Pienaar and Turnbull (1973) as a basis for basal area prediction in unthinned and thinned even-aged coniferous monocultures for a given site class. The first coefficient was expanded to be a function of site index as follows.

$$(12) \quad \frac{dBA}{dt} = (g_1 + g_2 \cdot SI) \cdot BA^{g_3} - g_4 \cdot BA$$

4.3.2 Top height

Height growth was predicted using the same site index equations as in section 4.2.2. The site index was computed from the height and age at the beginning of the growth period and then the age incremented to predict the height at the end of the growth interval.

4.3.3 Density

For plantations, the target is often specified in terms of stems/ha rather than basal area so density is a relatively important attribute. The following model form was selected to predict the relative change in density.

$$(13) \quad \frac{dN}{N} = h_1 + h_2 \cdot age + h_3 \cdot BA$$

N is density, age is stand age or years from planting and BA is basal area. There was a tendency for density to increase in the first 20 years due to ingrowth and upgrowth (trees crossing the dbh measurement threshold). This was particularly apparent in the

plantations, possibly because they tended to be younger. In any case, the density change model is intended to predict stem survival and is a non-increasing function. Therefore, for plantations, the maximum stems/ha were used as the density estimate until mortality began and then the actual density was used.

4.3.4 Volume

Volume growth was predicted using the following equation.

$$(14) \quad \frac{dVol}{dt} = Vol \left[\frac{dH/dt}{H} + \frac{dBA/dt}{BA} \right]$$

This equation assumes the stand form factor (ratio between volume and BA×H) is constant over time.

The merchantability and cull factors from section 4.2.4 were used to estimate gross merchantable and net merchantable volumes.

4.4 Genetic Gain

Tree improvement programs have been underway in Ontario for many years and improved stock is available for black spruce and for jack pine. The genetic gain has been incorporated as an increase in site index.

4.5 Thinning response

Little data were available from thinned stands. Thinning is not recommended in the silvicultural guides for the forest units examined. It is generally accepted that per area growth of thinned stands is slower than unthinned stands immediately after thinning. Thinning may eventually increase volume growth and standing volume per area provided the residual trees can refill the growing space rapidly (Oliver and Larson 1996 p.350). In the absence of thinning guidelines and substantial data, thinning is implemented in the yield curves as a “parallel response” in terms of basal area growth. That is, following thinning, basal area grows at the same rate as an unthinned stand resulting in a parallel yield curve for basal area. Top height development is assumed to be independent of thinning.

4.6 Mixed species forest units

Some of the forest units do not have a single dominant species. For example, in the Romeo Malette Forest, the average species composition for the MW1 forest unit is Pj₃₄Po₃₁Bw₁₉Sb₁₁Sw₂Bf₂. If the MW1 is broken down by leading species, approximately 40% of the area in the MW1 forest unit has jack pine leading, 34% poplar, 20% white birch with minor areas in balsam fir, cedar, and black spruce. Generally the leading species is at least 40% of the species composition. In deriving the summary of the forest unit, SFMMTool averages the site classes, regardless of leading species, and assigns the average site class to the leading species of the average species composition.

As an artificial example, let half the forest unit be Pj₆₀Po₄₀ with a site class of 2 and the other half be Po₆₀Pj₄₀ with a site class of 1. The average species composition is Pj₅₀Po₅₀

with an average site class of 1.5. If the average site class is applied to the jack pine, it implies a dominant height of 16.8m at age 50. If the average site class is applied to poplar, it implies a dominant height of 21.6m at age 50.

Likewise stocking is dependent on which species is used as the working group.

The situation is more complicated because the site class is not necessarily based on the leading species of the stand. Where possible, future inventories should record the species for which the age, height, and site index were computed.

For forest units that have more than one working group represented by significant areas, the following approach is recommended. It is based on the assumption that the age, height, site index, and stocking attributes are based on the working group species. As noted in the previous paragraph, this is not always the case. However, it is beyond the scope of this project to solve that problem.

1. Calculate the forest unit summaries by leading species. Leading species with minor areas (<5% of the forest unit area or less than 100ha) may be combined with other leading species.
2. Derive yield curves for each leading species using the yield curves derived for the forest units with a single dominant species. For the artificial example above, a yield curve for the Pj₆₀Po₄₀ area would be derived from the PJ1 or PJ2 forest unit (as appropriate) and the yield curve for the Po₆₀Pj₄₀ portion derived from the PO1 curves. The two yield curves would then be averaged (weighted by area) to produce the mixed forest unit yield curve.

There are a number of advantages to this approach. It attempts to keep site index associated with a species. It also allows the leading species to be used when specifying management intensities. For example, for spruce/aspen mixes, an intensive forest management strategy may involve removing the aspen understory at age 60 and leaving the spruce for another 40 years before final removal, whereas intensive management in the jack pine/aspen mixes may involve early cleaning to remove the aspen and rehabilitate the site to a pine forest unit. In either case, the working group is important in determining the silvicultural prescription, not the working group average.

4.7 Confidence intervals

The general formula for variance associated with nonlinear (or linear) predictions using basal area as an example is as follows.

$$V(Y | x) = \frac{\partial Y}{\partial X} V(\alpha_0, \alpha_1, \beta_0, \gamma_0) \frac{\partial Y}{\partial X}^T$$

where

(15) $V(Y | x)$ = variance associated with predicting basal area for vector x

$\frac{\partial Y}{\partial X}$ = vector partial derivatives of the BA prediction equation with respect to X

$V(\alpha_0, \alpha_1, \beta_0, \gamma_0)$ = variance covariance matrix associated with the parameters of the BA prediction model

The variance is a function of the values of the independent variables, the parameter estimates and the variance covariance matrix of the parameters.

The confidence intervals associated with equation (15) were calculated as follows.

$$(16) \quad \hat{Y} \pm t_{(0.025, dfe)} \sqrt{\frac{V(Y|x)}{n}}$$

Total stem volume was calculated using equation (6). In this equation, the z_0 parameter and the predicted BA term have associated variances. The errors associated with top height were not quantified here and are not considered. The variance of a product of two random variables involves third order moments. For this project, the z_0 and BA terms were assumed to be independent, simplifying the covariance formula to the following.

$$(17) \quad Var(XY) = \mu^2_x Var(Y) + \mu^2_y Var(X) + Var(X)Var(Y)$$

Multiplying the product by the $topht$ (treated as a constant here) led to the following variance estimate.

$$(18) \quad Var(\hat{volume}) = topht^2 Var(z_0 \hat{BA})$$

For example, consider a 50 year old PJ1 stand with an average site index of 15m and stocking of 1.00. The estimate of top height from equation (4) is as follows.

$$(19) \quad \begin{aligned} topht &= 1.3 + 4.1459 \cdot (15 - 1.3)^{0.6224} \cdot (1 - K^{(50-6)/50})^{1.3723 \cdot (15-1.3)^{-0.0802}} = 13.948 \\ \text{where} \end{aligned}$$

$$K = 1 - \left(\frac{15 - 1.3}{4.1459 \cdot (15 - 1.3)^{0.6224}} \right)^{\frac{1}{1.3723 \cdot (15 - 1.3)^{-0.0802}}} = 0.318$$

The estimate of basal area is the following

$$(20) \quad \hat{BA} = 1.00 \cdot (9.5067 + 1.04388 \cdot 15) \cdot (1 - e^{-0.02210 \cdot 50^{1.28348}}) = 24.282$$

The estimated variance associated with the volume coefficient z_0 is the following.

$$(21) \quad Var(\hat{z}_0) = \frac{\hat{\sigma}^2}{\sum x \cdot x} = \frac{\hat{\sigma}^2}{\sum \hat{ba} \cdot topht \hat{ba} \cdot topht} = \frac{898.73}{274695331} = 0.00000327$$

Equation (15) becomes the following.

$$(22) \quad Var(\hat{ba}) = \begin{bmatrix} 2.543 & -0.169 & 0.00451 & -0.0579 \\ -0.169 & 0.0121 & -0.000187 & 0.00173 \\ 0.00451 & -0.000187 & 0.000137 & -0.00192 \\ -0.0579 & 0.00173 & -0.00191 & 0.0278 \end{bmatrix} \begin{bmatrix} 0.965 \\ 14.47 \\ 13382 \\ 11.57 \end{bmatrix} = 0.1327$$

This leads to an estimate of the total volume variance of the following.

$$(23) \quad Var(\hat{vol}) = 13.948^2 \cdot (0.4322^2 \cdot 0.1327 + 24.282^2 \cdot 0.00000327 + 0.1327 \cdot 0.00000327) = 5.20$$

This is associated with a volume estimate of the following.

$$(24) \quad \hat{vol} = \hat{z}_0 \cdot \hat{ba} \cdot topht = 0.4322 \cdot 24.282 \cdot 13.948 = 146.38$$

plantation with an initial density of 2000 stems/ha, a species composition of Pj₈₀Sb₁₀PO₁₀ and a SC = 1.7. “Intensive 1” represents a similar plantation but with the addition of a pre-commercial thinning to remove 1/3 of the basal area at age 20 and a commercial thinning to remove 1/3 of the basal area at age 35. “Intensive 2” is the same as “Intensive 1” but with a 3% genetic gain. “Intensive 3” is the same as “Intensive 1” but with an 8% genetic gain. “Present” represents the present management intensity from the current RFM management plan. Dashed lines indicated extrapolations beyond the age range of the data.

5.2 SP1

The results for the SP1 forest unit are compared to the curves in the current management plan in the following figure. The proposed natural curve (solid, heavy black) is similar to the curve used in the current RMF plan (RMF-present) up to about 110 years old but then the proposed curve is higher.

The plantations, again, show much higher growth, although the oldest plantations in the data set are 48.

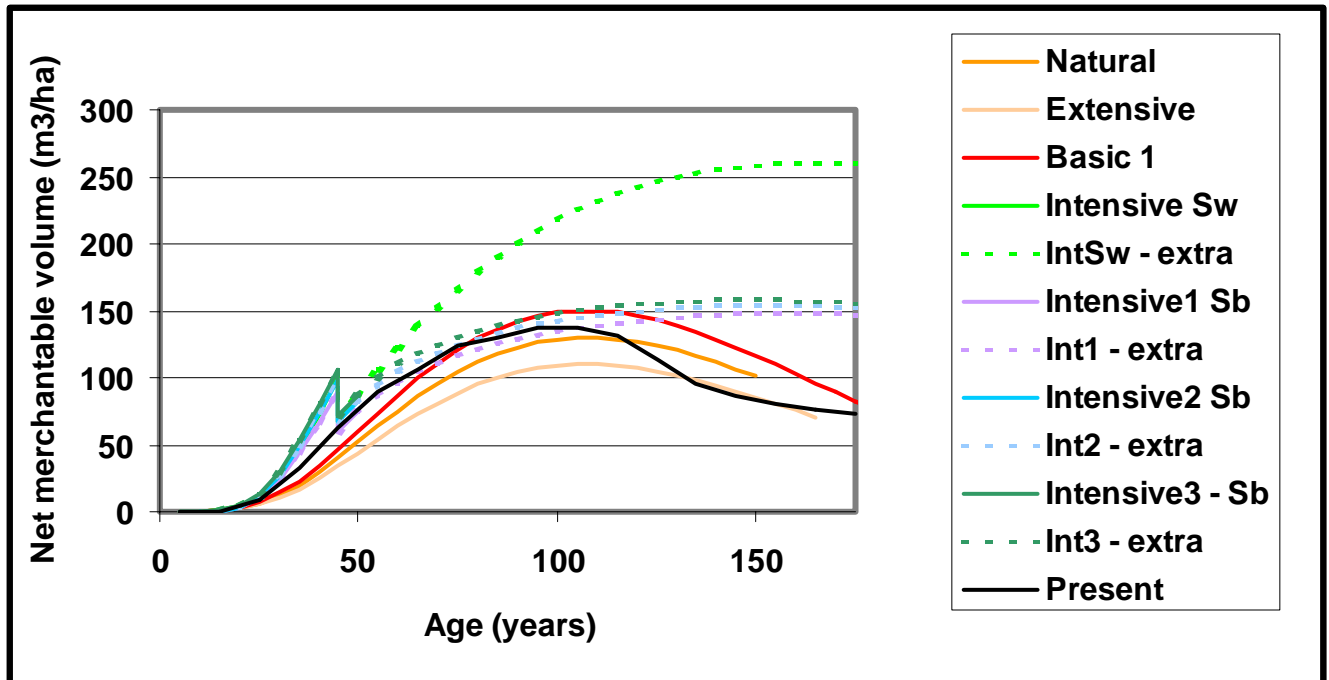


Figure 7. The yield curves for the SP1 forest unit within the Romeo Malette management unit. The management intensities are described in Table 2. “Natural” describes the current forest (Species composition $Sb_{60}Bw_{14}Po_9Pj_8Bf_6Ce_2Sw_2$, stocking given in Figure 5b, $SC = 1.1$). “Extensive” has the same attributes as natural but stocking is reduced by 0.1. “Basic” has the same attributes as natural but stocking is increased by 0.1. “Intensive Sw” represents a white spruce plantation with an initial density of 2000 stems/ha, a species composition of $Sw_{80}Pj_{10}Po_{10}$, a $SC = 1.4$ and a commercial thinning to remove 1/3 of the basal area at age 35. “Intensive 1 Sb” represents a similar plantation but with a species composition of $Sb_{80}Pj_{10}Sw_{10}$. “Intensive 2 Sb” is the same as “Intensive 1 Sb” but with a 9% genetic gain. “Intensive 3 Sb” is the same as “Intensive 1 Sb” but with a 15% genetic gain. “Present” represents the present management intensity from the current RMF management plan. Dashed lines indicate extrapolations beyond the age range of the data.

5.3 SB1

The results for the SB1 forest unit are compared to the curves in the current management plan in the following figure. The intensive management intensity includes a thinning and the predictions should be considered an extrapolation.

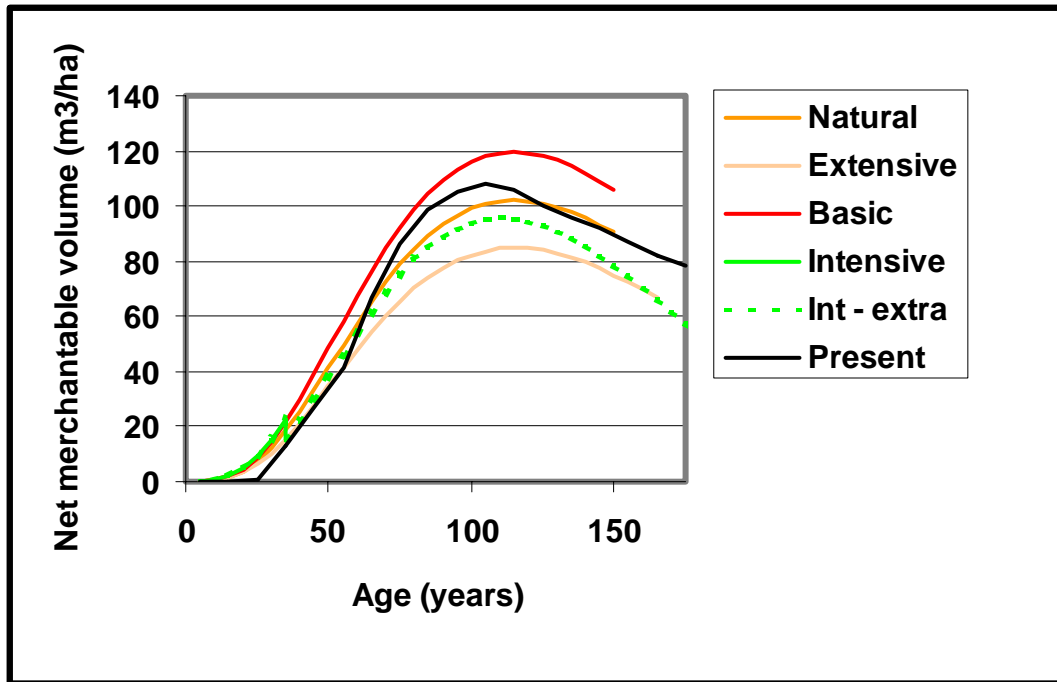


Figure 8. The yield curves for the SB1 forest unit within the Romeo Malette management unit are given for the management intensities described in Table 2. “Natural” describes the current forest (Species composition $Sb_{91}Ce_2La_2Po_2Bw_2$, stocking given in Figure 5c, $SC = 1.4$). “Extensive” has the same attributes as natural but stocking is reduced by 0.1. “Basic” has the same attributes as natural but stocking is increased by 0.1. “Intensive” has the same attributes as “Basic” with a commercial thinning to remove 1/3 of the basal area at age 35. “Present” represents the present management intensity from the current RFM management plan. Dashed lines indicate extrapolations beyond the age range of the data.

5.4 PO1

The results for the PO1 forest unit are compared to the curves in the current management plan in the following figure. The intensive management option includes a thinning and should be considered an extrapolation.

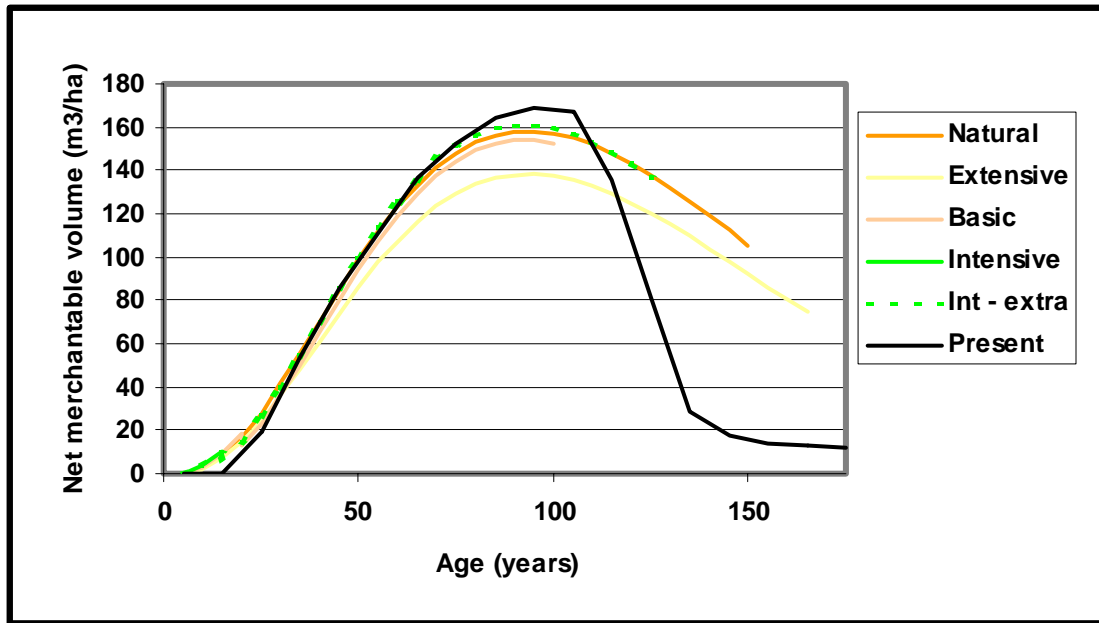


Figure 9. The yield curves for the PO1 forest unit within the Romeo Malette management unit are given for the management intensities described in Table 2. “Natural” describes the current forest (Species composition $P_{070}B_{w11}S_{b8}P_{j4}S_{w3}B_{f3}$ stocking given in Figure 5d, $SC = 1.7$). “Extensive” has the same attributes as natural but lower aspen and more fir (Species composition $P_{065}B_{f8}B_{w11}S_{b8}P_{j4}S_{w3}$ stocking 0.68 of natural, $SC = 1.7$). “Basic” has the same attributes as natural but higher aspen and more fir (Species composition $P_{080}B_{w6}S_{b5}P_{j4}S_{w3}B_{f3}$, stocking 0.88 of natural, $SC = 1.7$). “Intensive” has the same attributes as “Basic” with a thinning to remove 1/3 of the basal area at age 15. “Present” represents the present management intensity from the current RFM management plan. Dashed lines indicate extrapolations beyond the age range of the data.

5.5 SF1

The results for the SF1 forest unit are compared to the curves in the current management plan in the following figure. The intensive management option includes a thinning and should be considered an extrapolation.

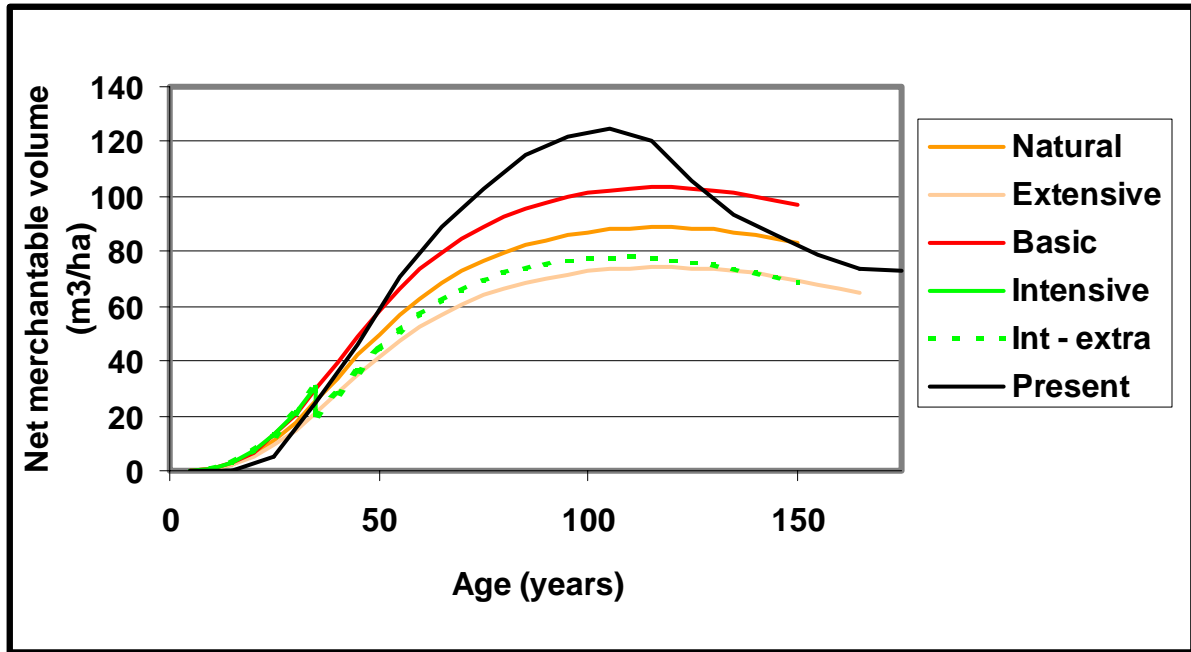


Figure 10. The yield curves for the SF1 forest unit within the Romeo Malette management unit are given for the management intensities described in Table 2. “Natural” describes the current forest (Species composition $Sb_{30}Sw_{18}Bf_{16}Bw_{15}Ce_{12}Po_7La_1$, stocking given in Figure 5e, $SC = 1.2$). “Extensive” has the same attributes as natural but stocking is reduced by 0.1. “Basic” has the same attributes as natural but stocking is increased by 0.1. “Intensive” has the same attributes as “Basic” with a commercial thinning to remove 1/3 of the basal area at age 35. “Present” represents the present management intensity from the current RFM management plan. Dashed lines indicate extrapolations beyond the age range of the data.

6 Discussion

Most of the forest units show a surprising amount of agreement between the “Present” curve, which represents the natural conditions from the current RFM management plan, and the “Natural” curve derived in this project.

The largest differences between the “Present” management intensity from the current RFM plan and the “Natural” management intensity developed in this project is for the PJ1 forest unit. This largely due to the small difference between gross total and gross

merchantable volume for the PJ1 forest unit and the relatively low cull up to age 100. The difference between the “Present” management intensity from the current RMF plan and the “Natural” management intensity developed in this project is also large for the SF1 forest unit. This is likely due to the very small sample size in Ontario and a result of the mixture of species in the forest unit.

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8 Appendix

8.1 Forest unit definitions

Table A1. The algorithm for assigning the plots to the forest units is given. The queries are sequential. Where additional information was available (e.g., ecotype, soils information, etc.), it was used to refine the assignment of plots to forest units. The Northeast Region (NER) forest unit algorithms are taken from Watt et al (2001). The Southcentral Region (SCR) forest units are taken from OMNR correspondence with a date of June 6, 2001.

REGION	FU Name	FU Description	Sort Order	Forest Unit query Syntax
NER	Pr1	Red Pine 1	1	$Pr \geq 0.7$
NER	Pw1	Pw Mixed 1	2	$Pw+Pr+Sw+He \geq 0.4$ and $Pw \geq 0.3$
NER	Prw	Pr Mixed	3	$Pw+Pr \geq 0.4$
NER	LH1	LowLand Hwd	4	$LH \geq 0.3$
NER	TH1	Tolerant Hwd	5	$LH+MH+UH \geq 0.3$
NER	BOG	Sb Bog	6	$Sb+La \geq 0.7$ and $Pw = 0$ and $SC = '4'$
NER	Sb1	Sb LowLand	7	$Sb \geq 0.8$ and $MH+UH+Pr = 0$ and $Pw+Pj \leq 0.1$
NER	Pj1	Jack Pine	8	$Pj \geq 0.7$ and $Po+Bw+MH+UH+LH \leq 0.2$
NER	LC1	Sb / Ce / La	9	$Ce+La+Sb \geq 0.8$ and $MH+UH+Pr = 0$ and $Pw+Pj \leq 0.1$
NER	Pj2	P j /Sb	10	$(Pj+Sb+Pr \geq 0.7$ or $(Pj \geq 0.5$ and $Pj+Sb+Bf+Sw+He+Pw+Pr+Ce+La \geq 0.7$ and $Bf+Sw+He+Pw+Ce+La \leq 0.2))$ and $Pj \geq Sb$
NER	SP1	Sb / Pj	11	$Sb+Sw+Bf+Ce+La+Pw+Pj+Pr+He \geq 0.7$ and $(Bf+Ce+Pw+La+Sw+He \leq 0.2$ or $Pj \geq 0.3)$
NER	SF1	Sp / Bf / Ce	12	$Sb+Sw+Bf+Ce+La+Pw+Pj+Pr+He \geq 0.7$
NER	Po1	Aspen	13	$Po+Bw+MH+UH+LH \geq 0.7$ and $Po \geq 0.5$
NER	Bw1	White Birch	14	$Po+Bw+MH+UH+LH \geq 0.7$
NER	Mw1	Pj / Bw / Po	15	$Pj+Pr \geq 0.2$
NER	Mw2	Sb / Po	16	What's Left
SCR	PW1	Pw-PwQr-US4	1	$Pw \geq Pr$ and $Pw \geq He$ and $Pw \geq Sw$ and $Pw \geq Pj$ and $Pw \geq Qr$ and $(Pw+Pr \geq .5$ and $Stk \geq .7$ or $Pw+Pr+He+Sw+Pj+Qr \geq .7)$
SCR	PW2	PwUS2	2	$Pw \geq Pr$ and $Pw \geq He$ and $Pw \geq Sw$ and $Pw \geq Qr$ and $Pw \geq Sb$ and $Pw \geq Ce$ and $Pw \geq Pj$ and $(Pw+Pr \geq .3$ and $Pr+Pw+He+Sw+Qr+Sb+Ce+Pj \geq .5$ and $Stk \geq .6$ or $Pw+Pr+He+Sw+Qr+Sb+Ce+Pj \geq .7)$
SCR	PR3	Pr Plantations	3	$Pr = 1$ and $stk > 0.8$ or $(Pr \geq 0.7$ and $(stype = 23$ or $stype = 28))$ (modify)
SCR	PR1	PrUS2	4	$Pr > Pw$ and $Pr \geq Sw$ and $Pr \geq Qr$ and $Pr \geq Pj$ and $Pr \geq Sb$ and $(Pr+Pw \geq .3$ and $Pr+Pw+Sw+Qr+Pj+Sb \geq .4$ and $Stk \geq .6$ or $Pr+Pw+Sw+Qr+Pj+Sb \geq .7)$
SCR	BY1	By US	5	$Yb \geq .4$
SCR	HE1	He Sel	6	$He \geq .5$
SCR	OR1	Qr-QrPine-US3	7	$Qr \geq Pw$ and $Qr \geq Pr$ and $Qr \geq He$ and $Qr \geq Sw$ and $Qr \geq .3$ and $Qr+Pw+Pr+He+Sw \geq .5$
SCR	LH1	Low Hdwd Sel	8	$Ce+La+Sb+Ab+Ew > .5$ and $Ab+Ew+Ms+Yb > Sb+Ce+La+Bf$

REGION	FU Name	FU Description	Sort Order	Forest Unit query Syntax
SCR	CE1	Ce Low	9	Ce+La+Sb+Ab+Ew>.5 and Ce>Sb+La+Bf
SCR	OC1	Other Con Low	10	Ce+La+Sb+Ab+Ew>.5 and Sb+La+Bf>=Ce
SCR	HD1	Tol Hdwd Sel	11	Mh+Ab+Aw+Bd+Be+Ch+Ew+lw+Qr+Yb+Ow+Ob+He>=.5 and Po+Bw+Bf<=.3 and (Sc = 'X' OR Sc = '1' OR Sc = '2') and Age>=80 and Stk>=.6
SCR	HD2	Tol Hd Fut Sel	12	Mh+Ab+Aw+Bd+Be+Ch+Ew+lw+Qr+Yb+Ow+Ob+He>=.5 and Po+Bw+Bf<=.3 and (Sc = 'X' OR Sc = '1' OR Sc = '2') and (Age<80 and Stk>=.6 or Age>=80 and Stk<.6)
SCR	HD3	Tol Hdwd US	13	Mh+Ab+Aw+Bd+Be+Ch+Ew+lw+Qr+Yb+Ow+Ob+He>=.5
SCR	CM1	Con Upd	14	Pw+Pr+Pj+Ps+Sb+Sw+Ce+La+Oc+He+Bf>=.7
SCR	PO1	Poplar 1	15	Po>=.7
SCR	INT	Poplar/Bw	16	Po+Bw+Ms>=.7
SCR	MW1	Mixedwood	17	FU='---'

8.2 Data

The raw data are plotted by forest unit in Figure 11 through Figure 18. As noted in the text, in general the sample plots have not been randomly located. In the past, sample plots tended to be located in pure conifer stands on better sites with high stocking levels and excellent vigour and health. Plantations tend to be confined to the better sites as well. The current Ontario growth and yield program targets specific conditions (e.g., low density, high site class, immature stands) for sample plot locations. Therefore, in general, the data in used in this project do not represent a random sample, nor can an average line fit to the data be interpreted as the average yield of the forest unit.

The modeling process addressed some of the biases in the plot locations by including site index, stand origin, and stocking as covariates. If the data in the following graphs were plotted by site index and stocking class, the variation would be considerably reduced.

8.2.1 PJ1

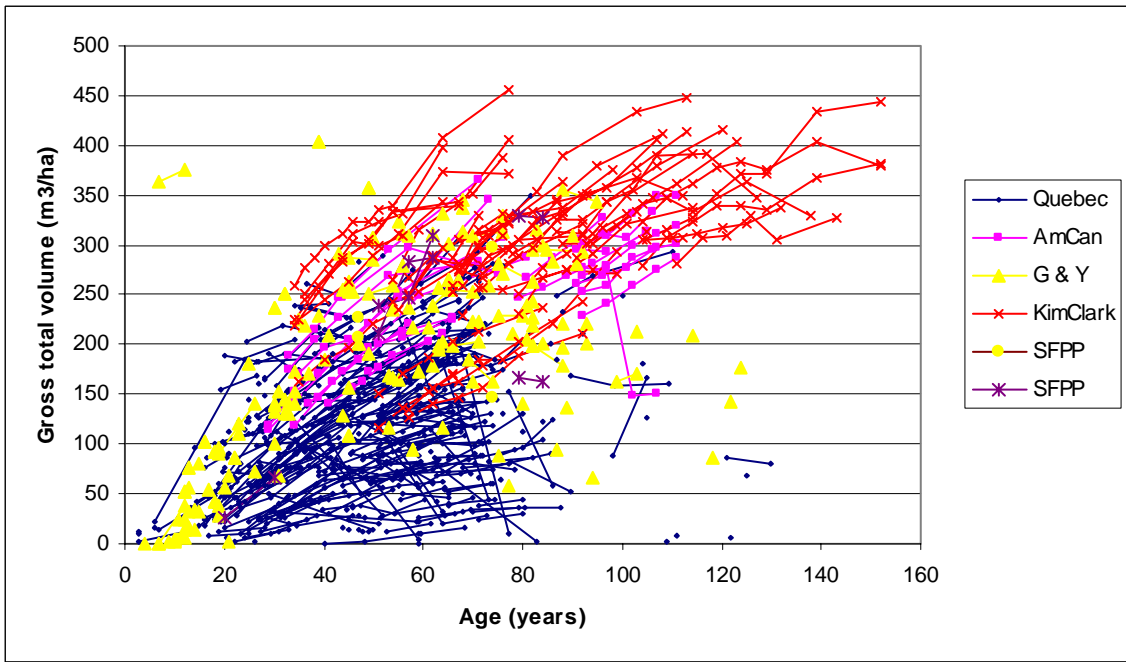


Figure 11. The data for natural stands in the PJ1 forest unit are plotted by data source.

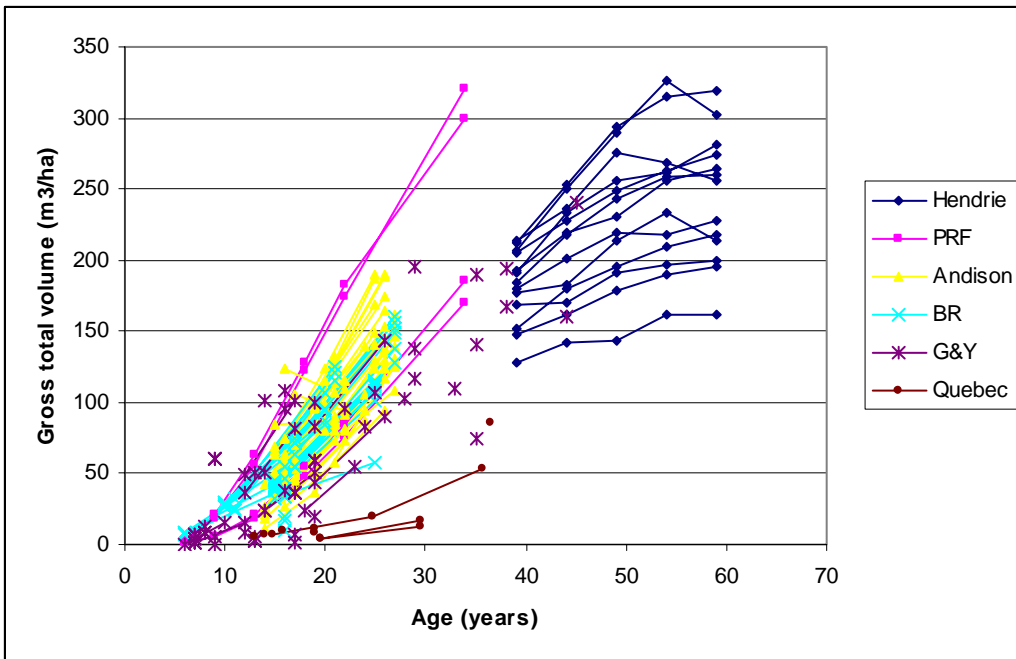


Figure 12. The data for plantations in the PJ1 forest unit are shown by data source. The Hendrie plantation was near Barrie, ON.

8.2.2 SP1

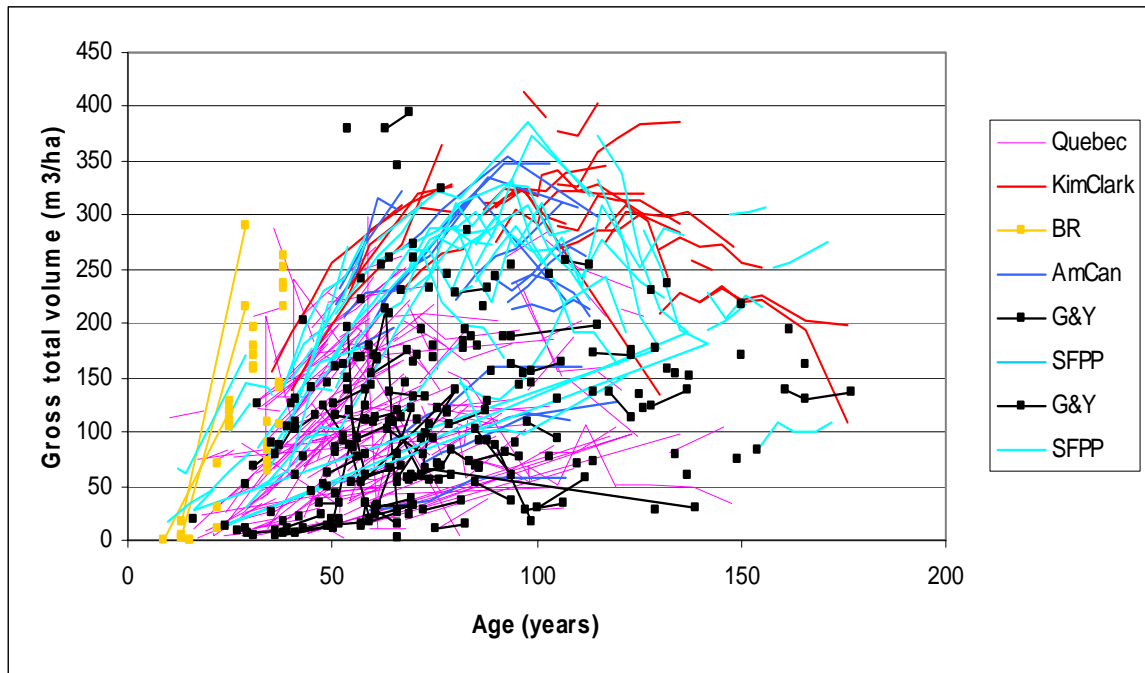


Figure 13. The data for natural-origin plots within the SP1 forest unit are plotted.

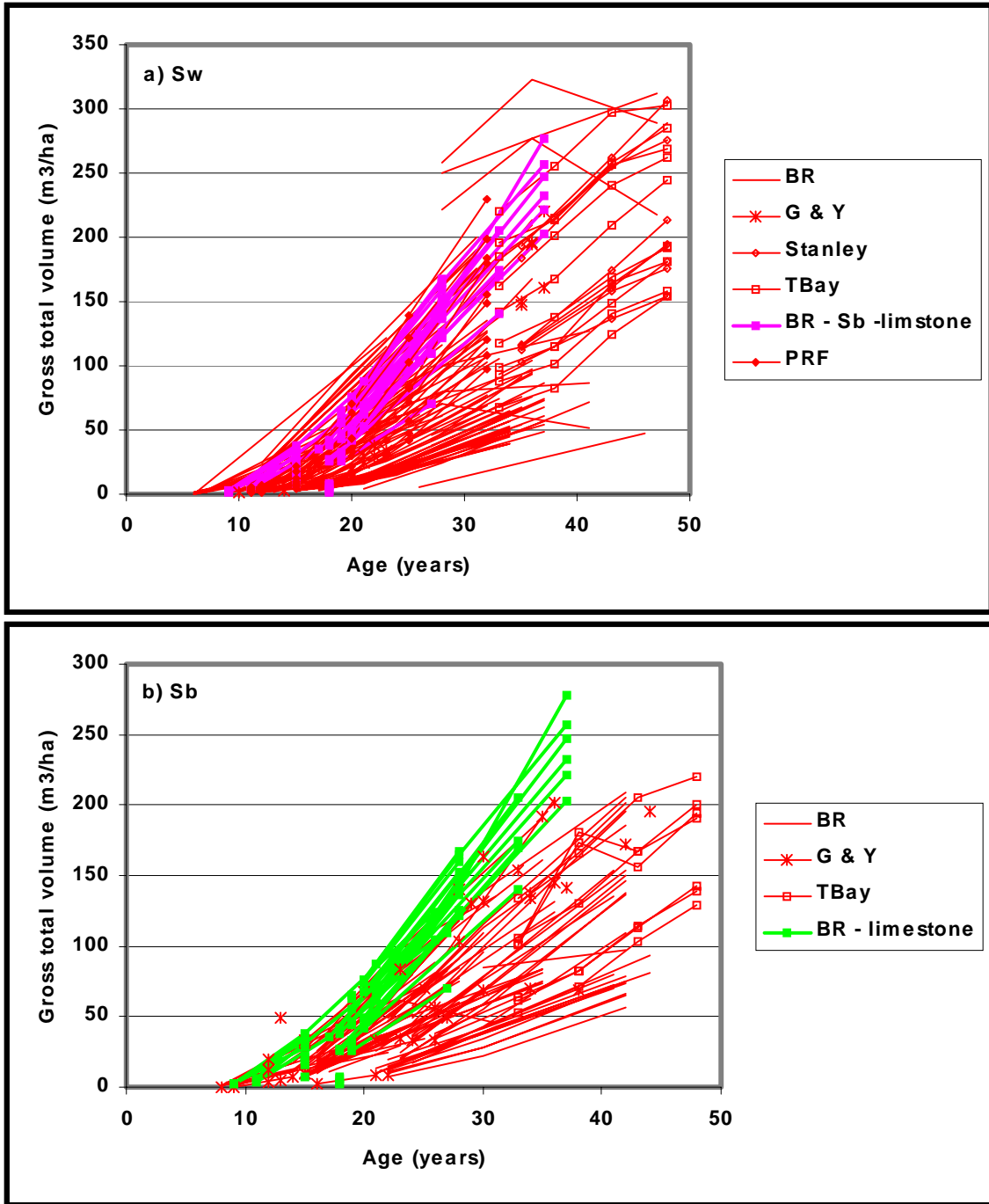


Figure 14. The data for plantation-origin plots within the SP1 forest unit are plotted. White spruce plantations are given in figure a) and black spruce plantations are given in figure b). The limestone plots were removed before any of the analysis.

8.2.3 SB1

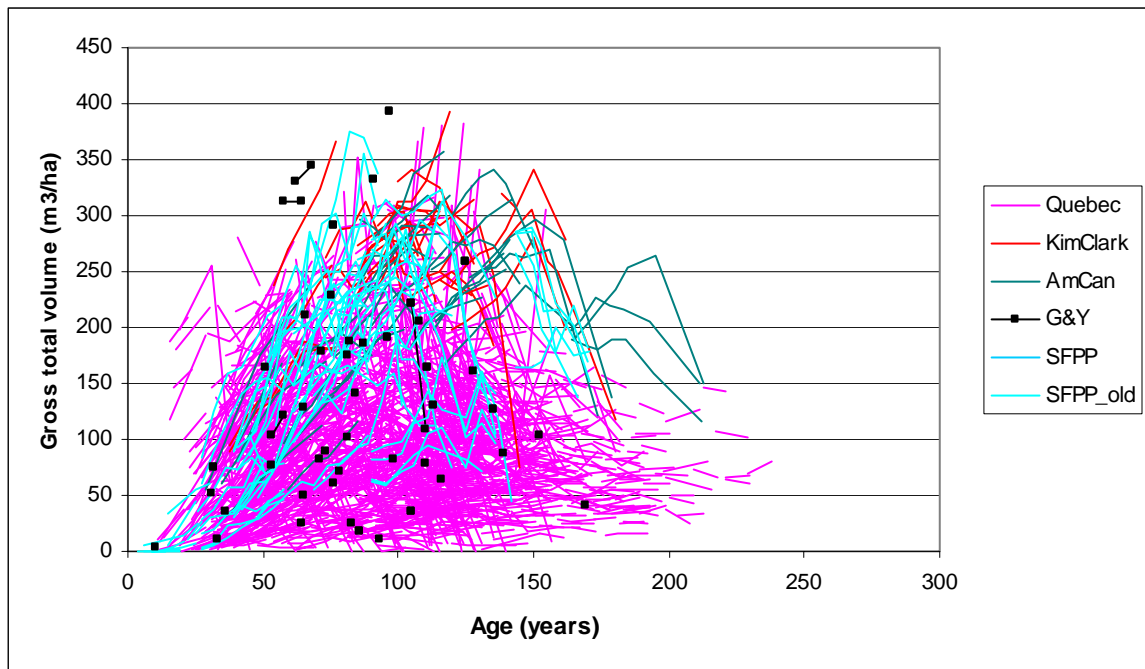


Figure 15. The data for natural-origin plots within the SB1 forest unit are plotted.

8.2.4 PO1

The data for the PO1 unit are given in the following figure. Note the significant volumes at young ages. For cut-origin stands, the stand age is calculated from the year of harvesting. Some of these stands may have a significant component of residual hardwoods left following harvesting. These residual trees may not survive to the next rotation.

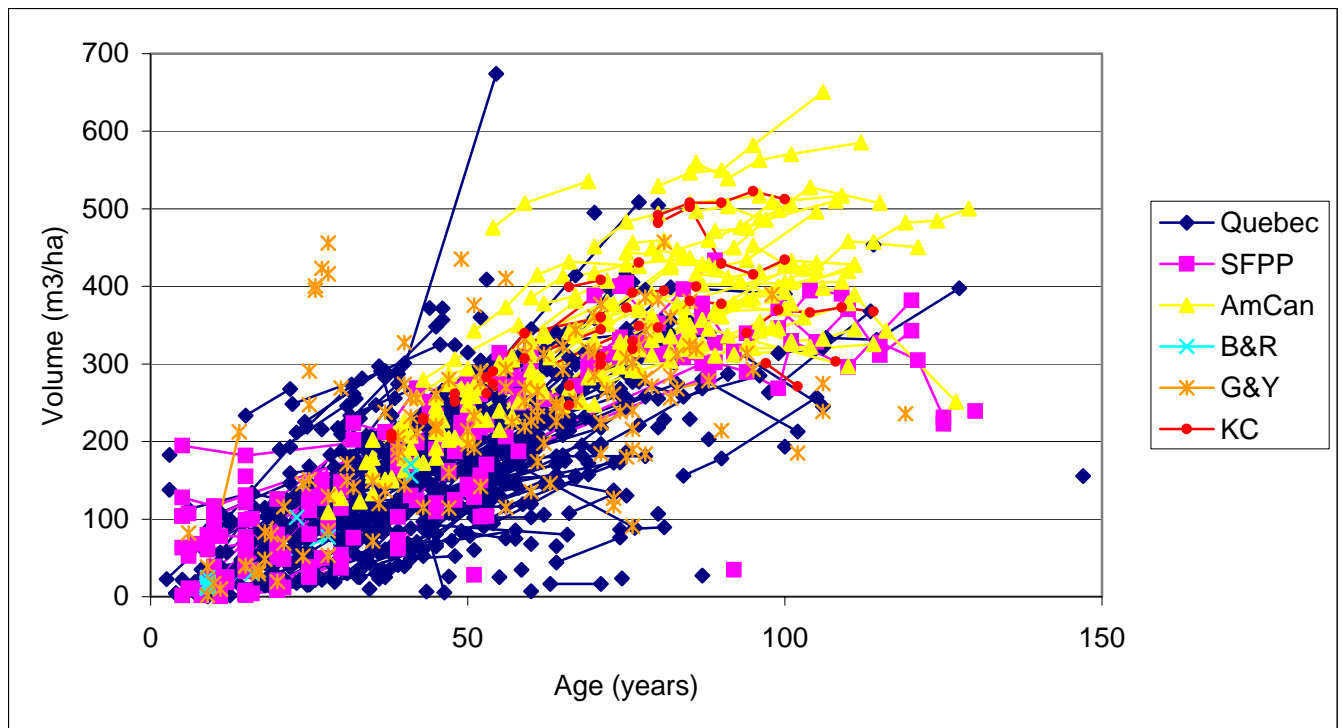


Figure 16. The data associated with the PO1 forest unit are given by data source.

8.2.5 SF1

The data for the SF1 forest unit are given in the following figures.

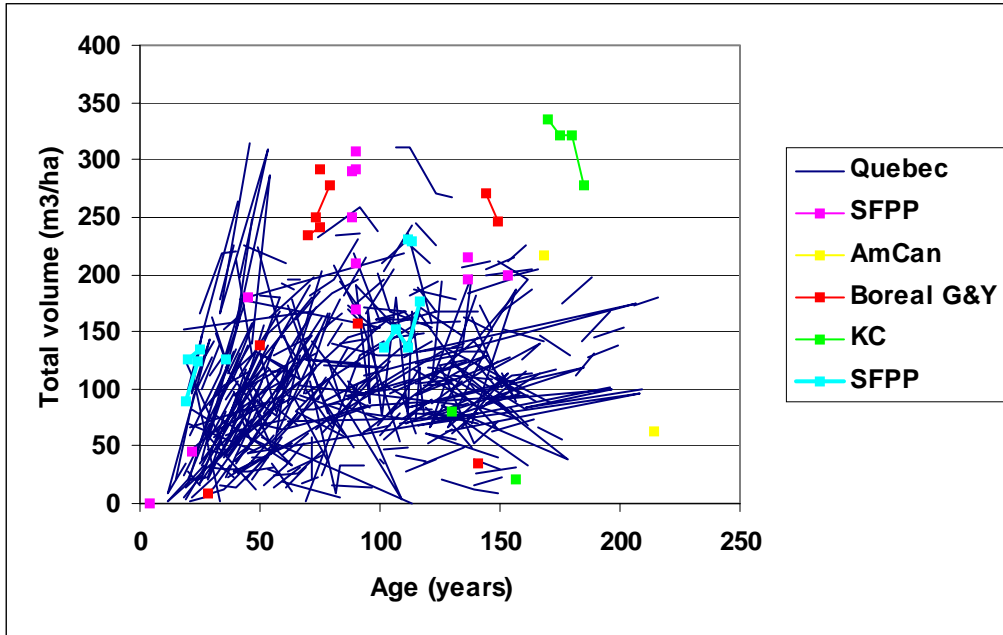


Figure 17. The data associated with the SF1 forest unit are given by data source.

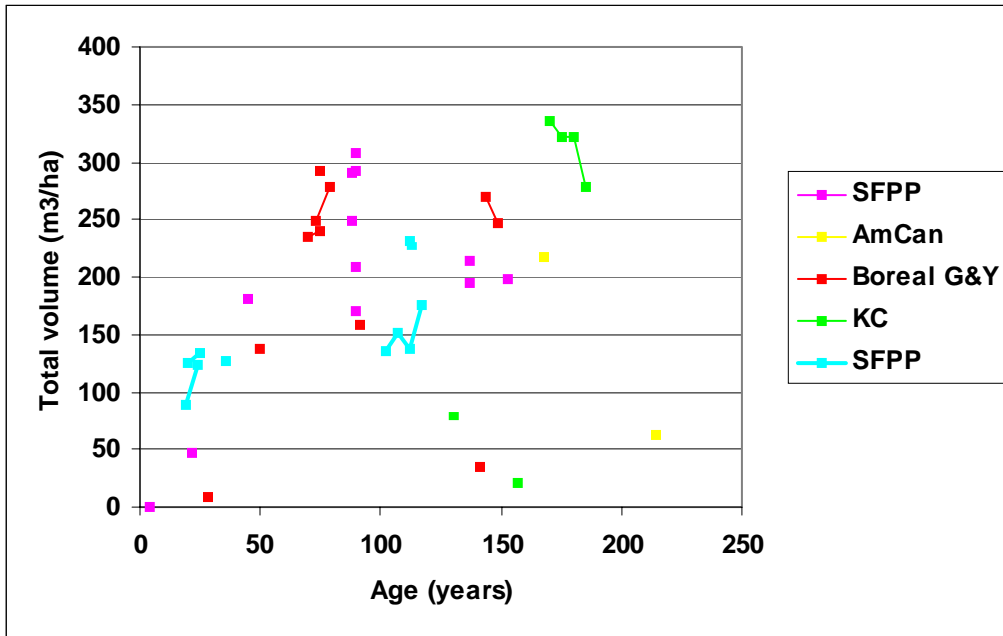


Figure 18. The same as Figure 17 with the Quebec data removed.

8.3 Statistical fits

The statistical fits are given for the main pine, poplar and spruce forest units. The PJ2 forest unit is given here although it was not discussed in the text.

The stocking equations are given in the following table.

Table 6. The fits for the stocking equation (2) are given by forest unit.

FU	N	b_0	b_1	b_2	R^2	Average stocking	Mean resid	Mean squared residual
PJ1	104	75.185* ⁵	85.323*	204.74*	0.98	0.86792	-0.05040	0.06859
PJ2	103	69.896*	75.961*	164.68*	0.96	0.81456	0.00991	0.05504
PO1	188	54.301*	91.812*	193.44*	0.96	0.73511	-0.03331	0.05684
SB1	282	55.270*	104.620*	181.96*	0.94	0.59726	-0.01855	0.04529
SF1	209	19.686	161.725	277.63	0.94	0.61292	0.00092	0.05147
SP1	214	66.016*	79.485*	141.28*	0.95	0.61075	-0.01681	0.04987

The basal area prediction equations are given in the following table. The mean squared residual gives an idea of the fit of the equation but does not provide enough information to calculate the confidence around basal area predictions so the variance covariance matrix is provided in Table 8.

Table 7. The fits for the prediction equation (3) are given by forest unit.

FU	Origin	N	α_0	α_1	β_0	γ_0	R^2	Average Basal area (m ² /ha)	Mean resid	Mean squared residual
PJ1	N	1252	9.5067*	1.04388*	0.02210*	1.28348*	0.99	22.9345	0.00325	0.099
	P	341	10.2929	7.51906*	0.00100*	1.56033*	0.90	16.5315	0.56873	33.102
PJ2	N	356	9.3950*	1.06108*	0.03124*	1.17853*	0.99	25.3519	-0.01664	0.030
	P	8	0.0000	5.32122	0.00100	1.57663	0.95	17.6343	0.73466	20.678
PO1	N	1404	15.8998*	0.97722*	0.01286*	1.22651*	0.99	26.4338	0.00481	0.043
	P	4	1.5742	0.00000	1.14008	2.10160	0.99	4.8286	0.18751	0.309
SB1	N	4856	37.7106*	0.00000*	0.00100*	2.08785*	0.99	20.2874	0.28823	6.280
SF1	N	1159	23.6981*	1.57934*	0.00583*	1.37027*	0.99	22.8409	0.19951	8.618
SP1	B	276	25.8306*	1.16174	0.00100*	1.95807*	0.75	14.6614	2.30071	78.541
	N	1407	14.6151*	2.03553*	0.00100*	1.89327*	0.98	24.2540	0.56535	15.335
	W	424	0.0000	2.65764*	0.00100	1.85942*	0.72	16.2301	3.04372	127.909

⁵ An asterisk indicated the coefficient is statistically significant at the $\alpha=0.05$ level.

Table 8. The variance covariance matrix for the basal area predictions is given by forest unit and origin.

FU	Origin	N	α_0	α_1	β_0	γ_0
PJ1	N	1252	2.5427	-0.1691	0.0045	-0.0579
			-0.1691	0.0121	-0.0002	0.0017
			0.0045	-0.0002	0.0001	-0.0019
			-0.0579	0.0017	-0.0019	0.0278
P	341	15401.7578	1844.2193	1844.2193	0.0000	-113.4120
			1844.2193	2679.4231	0.0000	-115.5618
			0.0000	0.0000	0.0000	0.0000
			-113.4120	-115.5618	0.0000	5.0777
PJ2	N	356	0.7174	-0.0504	0.0041	-0.0377
			-0.0504	0.0038	-0.0002	0.0014
			0.0041	-0.0002	0.0003	-0.0028
			-0.0377	0.0014	-0.0028	0.0263
PO1	N	1404	2.4227	-0.0954	0.0018	-0.0460
			-0.0954	0.0048	0.0000	0.0007
			0.0018	0.0000	0.0000	-0.0001
			-0.0460	0.0007	-0.0001	0.0036
SB1	N	4856	223.8741	-23.2523	0.0954	-3.3996
			-23.2523	3.3133	0.0004	-0.2708
			0.0954	0.0004	0.0006	-0.0266
			-3.3996	-0.2708	-0.0266	1.2689
SF1	N	1159	294.1216	-28.8613	0.0000	2.8046
			-28.8613	3.1364	0.0000	-0.3471
			0.0000	0.0000	0.0000	0.0000
			2.8046	-0.3471	0.0000	0.0605
SP1	B	276	0.0000	0.0000	0.0000	0.0000
			0.0000	233.4488	0.0000	-19.9935
			0.0000	0.0000	0.0000	0.0000
			0.0000	-19.9935	0.0000	1.7537
N	1407	739.1740	-66.6040	-66.6040	0.0000	2.7795
			-66.6040	6.5008	0.0000	-0.3660
			0.0000	0.0000	0.0000	0.0000
			2.7795	-0.3660	0.0000	0.0590
W	424	0.0000	0.0000	0.0000	0.0000	0.0000
			0.0000	155.2968	0.0000	-24.1501
			0.0000	0.0000	0.0000	0.0000
			0.0000	-24.1501	0.0000	3.8220

The stem number prediction equations are given in the following table.

Table 9. The fits for the prediction equation (5) are given by forest unit.

FU	Origin	N	x_0	x_1	x_2	R^2	Average density (stems/ha)	Mean resid	Mean squared residual
PJ1	N	1250	40616.19*	0.06886	0.86341*	0.65	2170.65	55.54	768273
	P	337	276703.87*	0.00000	1.51018*	0.99	2394.63	30.08	5452
PJ2	N	356	37426.42*	0.27934*	0.76971*	0.71	2736.97	-33.85	385924
	P	8	436244.70	0.00000	1.52800	0.99	3437.50	12.41	2831
PO1	N	1402	391652.17*	0.25142*	1.29868*	0.66	3294.32	-172.71	258855
	P	4	40287.81	0.00000	0.13739	0.90	9800.00	1073.72	3983771
SB1	N	4846	248190.20*	0.00000	1.74607	0.70	3573.77	3310.23	17719612
SF1	N	1157	109554.47*	0.16975*	0.93094*	0.66	4469.94	171.89	10459152
SP1	B	275	372150.26*	0.10309*	1.50842	0.97	2768.83	82.01	268766
	N	1395	149678.24*	0.28798*	1.07287*	0.69	3460.04	-127.32	6534578
	W	423	461632.81*	0.00000	1.59096	0.99	2509.64	196.51	135454

The total volume prediction equations are given in the following table.

Table 10. The fits for the prediction equation (6) are given by forest unit.

FU	Origin	N	z_0	R^2	Average Total volume (m^3/ha)	Mean resid	Mean squared residual
PJ1	N	1250	0.43218*	0.98	172.505	4.7904	898.73
	P	338	0.46129*	0.99	104.370	1.8153	33.59
PJ2	N	356	0.40247*	0.99	180.090	3.6392	281.54
	P	8	0.44858*	0.99	102.194	0.9238	43.79
PO1	N	1402	0.37211*	0.99	213.179	1.0307	467.44
	P	3	0.27978*	0.99	25.353	1.4046	6.95
SB1	N	4846	0.37263*	0.95	111.091	1.1078	862.76
SF1	N	1157	0.33330*	0.99	120.473	2.4909	268.00
SP1	B	275	0.44647*	0.99	63.935	1.9089	79.25
	N	1395	0.40626*	0.98	152.607	0.8559	634.07
	W	423	0.37586*	0.99	78.746	4.2300	164.25

The gross merchantable volume prediction equations are given in the following table.

Table 11. The fits for the prediction equation (7) are given by forest unit.

FU	Origin	N	z_1	z_2	z_3	R^2	Average merchantable volume (m ³ /ha)	Mean resid	Mean squared residual
PJ1	N	1248	0.56730	0.737	1.00000	0.93	123.731	-31.0500	1825.68
	P	337	0.01149	1.375*	1.00000	0.83	61.341	4.3423	1655.65
PJ2	N	356	0.03559	0.952*	0.89257*	0.97	127.121	0.7788	775.71
	P	8	0.01253	1.197	1.00000	0.95	43.109	-1.1406	163.21
PO1	N	1402	0.12894*	0.521*	1.00000	0.93	127.728	-13.1363	2404.81
	P	3	0.00000	4.687	0.23451	0.95	4.689	-0.7946	3.38
SB1	N	4809	0.03299*	0.990*	0.70017*	0.90	65.561	-4.6141	774.83
SF1	N	1153	0.34444*	0.270*	1.00000	0.93	77.580	-1.9809	606.35
SP1	B	275	0.00000	3.850*	1.00000	0.58	30.280	13.4815	1068.07
	N	1394	0.02662*	0.875*	1.00000	0.93	99.246	-0.5174	1091.82
	W	423	0.00015	2.559*	1.00000	0.67	44.155	9.5374	1820.82

The cull prediction equations are given in the following table.

Table 12. The fits for the prediction equation (8) are given by species.

Species	N	d_0	d_1	R^2	Average cull (%)	Mean resid	Mean squared residual
White pine	9	0.004286	2.540	0.92	12.6333	0.05741	19.0960
Red pine	8	0.000101	0.983*	0.98	1.0625	-0.01634	0.0217
Jack pine	8	0.012639*	8.375*	0.99	11.6375	0.15603	0.5136
White spruce	7	0.032612*	450.219	0.93	4.2143	1.26605	3.1910
Black spruce	9	0.000387	1.157*	0.93	2.5222	-0.04757	0.5834
Hemlock	10	0.000825*	1.093*	0.99	10.1800	0.03403	1.6121
Balsam fir	7	0.003204	1.558*	0.96	11.8429	-0.05868	7.5395
Cedar	6	0.008940*	4.600*	0.99	13.5000	-0.21733	1.9367
Hard maple	11	0.000324*	0.411*	0.99	27.7364	-0.01493	1.9147
Soft maple	6	0.004848*	0.973*	0.99	38.4833	-0.06515	11.6006
Yellow birch	32	0.004459*	3.459*	0.94	14.7094	0.68286	18.0453
White birch	7	0.003290	1.597*	0.95	11.4429	-0.29663	9.5904
Red Oak	4	0.003232*	2.138*	0.99	6.4500	0.06837	0.3124
Beech	7	0.002485	0.883*	0.99	27.7429	0.00328	10.0369
Black ash	6	0.027428*	12.299	0.98	42.8833	0.13151	44.9014
Elm	4	0.021696	9.843	0.95	19.0000	1.15546	29.1463
Basswood	8	0.007164*	3.824*	0.97	11.6000	0.37396	8.2475
Ironwood	5	0.003747	0.909	0.97	30.6600	-0.21945	33.0685
Aspen	7	0.005207*	1.405*	0.99	24.8714	0.07558	1.1506
131	16	0.003614*	3.132*	0.99	3.5500	-0.03973	0.0199
132	16	0.002276*	2.961*	0.99	1.3438	-0.05174	0.0525
Lowland hardwood	16	0.015715*	4.068*	0.93	35.2625	0.11479	115.794
Upland hardwood	56	0.000682	0.739*	0.75	16.7286	-0.01288	107.650

The product fraction prediction equations are given in the following tables.

Table 13. The fits for the prediction equation (10) for 16' SPF1 are given by forest unit.

FU	Origin	N	p ₁₁	p ₁₂	p ₁₃	p ₁₄	R ²	Average SPF1 volume (m ³ /ha)	Mean resid	Mean squared residual
PJ1	N	267	0.0000	0.03781	0.09777	0.67397	0.89	0.38066	-0.00510	0.0184
	P	68	0.6331	0.06186	0.00100	1.89965	0.83	0.27857	-0.03699	0.0217
PJ2	N	73	0.0000	0.03689	0.00100	2.03120*	0.92	0.42543	-0.00228	0.0167
	P									
PO1	N	297	0.6331*	0.00408	0.00100	2.42954*	0.56	0.25627	-0.00136	0.0530
	P									
SB1	N	248	0.0000	0.02881	0.12773	0.59102	0.82	0.25478	-0.00504	0.0151
SF1	N	78	0.1812	0.02605*	0.00100	2.41746*	0.92	0.45538	-0.00408	0.0205
SP1	B	22	0.0000	0.08627	0.02290	0.67589	0.73	0.16271	-0.01682	0.0119
	N	172	0.3434	0.07218	0.00100	1.90451	0.88	0.37424	-0.00096	0.0197
	W	60	0.0000	0.07363	0.00100	1.88765	0.91	0.27680	-0.02707	0.0102

Table 14. The fits for the prediction equation (10) for 8' SPF1 are given by forest unit.

FU	Origin	N	p ₂₁	p ₂₂	p ₂₃	p ₂₄	R ²	Average SPF1 volume (m ³ /ha)	Mean resid	Mean squared residual
PJ1	N	267	0.23109	0.00490	0.00100	1.93063	0.71	0.07835	-0.00307	0.0028
	P	68	0.00000	0.01492	0.00100	1.90416	0.73	0.06928	-0.00770	0.0024
PJ2	N	73	1.31475	-0.04138	0.00100	1.93508	0.77	0.11239	-0.00339	0.0046
	P									
PO1	N	297	0.12475	-0.00039	0.00301	1.91175	0.39	0.06813	0.00004	0.0073
	P									
SB1	N	248	0.00000	0.00752	0.05126	0.83224	0.66	0.05696	-0.00085	0.0018
SF1	N	78	0.56025	-0.00792	0.00100	2.01829*	0.77	0.13881	-0.00125	0.0063
SP1	B	22	0.80955	-0.04682	0.00100	1.98541	0.80	0.04444	-0.00147	0.0006
	N	172	0.00000	0.01348	0.01933	1.06303	0.70	0.08415	-0.00169	0.0032
	W	60	1.44145	-0.05042	0.00100	1.90750	0.92	0.17137	-0.00963	0.0031

Table 15. The fits for the prediction equation (10) for SPF2 are given by forest unit.

FU	Origin	N	p ₃₁	p ₃₂	p ₃₃	p ₃₄	R ²	Average SPF1 volume (m ³ /ha)	Mean resid	Mean squared residual
PJ1	N	267	0.23109	0.00490*	0.00100	1.93063	0.71	0.07835	-0.00307	0.0028
	P	68	0.00000	0.01492*	0.00100	1.90416	0.73	0.06928	-0.00770	0.0024
PJ2	N	73	1.31475	-0.04138*	0.00100	1.93508	0.77	0.11239	-0.00339	0.0046
	P									
PO1	N	297	0.12475	-0.00039*	0.00301	1.91175	0.39	0.06813	0.00004	0.0073
	P									
SB1	N	248	0.00000	0.00752*	0.05126	0.83224	0.66	0.05696	-0.00085	0.0018
SF1	N	78	0.56025*	-0.00792*	0.00100	2.01829	0.77	0.13881	-0.00125	0.0063
SP1	B	22	0.80955	-0.04682	0.00100	1.98541	0.80	0.04444	-0.00147	0.0006
	N	172	0.00000	0.01348*	0.01933	1.06303	0.70	0.08415	-0.00169	0.0032
	W	60	1.44145	-0.05042*	0.00100	1.90750	0.92	0.17137	-0.00963	0.0031

The basal area growth equation is given in the following table.

Table 16. The fits for the prediction equation (12) for basal area growth are given by forest unit.

FU	Origin	N	g ₁	g ₂	g ₃	g ₄	R ²	Average BA growth (m ³ /ha)	Mean resid	Mean squared residual
PJ1	N	648	0.12046	0.00270	0.74285*	0.06111	0.17	0.17019	-0.01642	0.18551
	P	156	0.53936*	0.03796	0.25090*	0.09676*	0.52	0.79739	0.06988	0.78184
PJ2	N	201	0.00000	0.03230*	0.18250	0.02657*	0.21	0.05084	-0.02720	0.26993
	P	3	1.12017	0.00115	0.93739	0.91257	N/A	0.50449	0.00000	0.00000
PO1	N	752	0.34799*	0.00665*	0.22120*	0.02762*	0.21	0.25046	0.03431	0.47137
	P	1	1.49983	0.07000	0.60000	1.00000	N/A	1.46759	-0.01747	0.00031
SB1	N	2753	0.24604*	0.00255	0.25108*	0.02343*	0.07	0.01000	-0.06653	0.33894
SF1	N	486	0.29597*	0.00506	0.41197*	0.04963*	0.24	0.13767	-0.01798	0.31331
SP1	B	156	1.36699	0.00000	0.92888*	1.06407	0.65	0.86189	0.11411	0.37184
	N	834	1.04968	0.00000	0.97129	0.94846	0.02	0.11672	0.00308	1.79227
	W	285	0.74887*	0.00087	0.33568*	0.05852*	0.76	0.91135	0.11016	0.29974

Table 17. The variance covariance matrix associated with the coefficients in equation (12) for basal area growth are given by forest unit.

FU	Origin	N	g ₁	g ₂	g ₃	g ₄
PJ1	N	648	78.1771	1.7544	482.1112	39.6584
			4.2246	-0.1441	15.5506	4.3983
			-0.1441	0.0053	-0.5626	-0.1547
			15.5506	-0.5626	60.9096	16.8324
	P	156	10.7270	-0.7948	-0.5844	-0.5152
			-0.7948	0.0769	-0.1035	0.0162
			-0.5844	-0.1035	2.2700	0.5840
			-0.5152	0.0162	0.5840	0.1898
PJ2	N	201	0.0000	0.0000	0.0000	0.0000
			0.0000	0.0216	-0.2536	-0.0055
			0.0000	-0.2536	3.7228	0.1283
			0.0000	-0.0055	0.1283	0.0072
PO1	N	752	4.7429	-0.1132	-1.3317	0.0181
			-0.1132	0.0073	-0.0937	-0.0079
			-1.3317	-0.0937	5.3017	0.3589
			0.0181	-0.0079	0.3589	0.0298
SB1	N	2753	5.5632	-0.0205	-10.5897	-0.3297
			-0.0205	0.0106	-0.2084	-0.0085
			-10.5897	-0.2084	31.7960	1.2657
			-0.3297	-0.0085	1.2657	0.0580
SF1	N	486	1.9290	0.0606	-4.9401	-0.4470
			0.0606	0.0051	-0.2828	-0.0298
			-4.9401	-0.2828	19.1807	2.0142
			-0.4470	-0.0298	2.0142	0.2222
SP1	B	156	2405.9890	0.0000	154.9015	2455.5099
			0.0000	0.0000	0.0000	0.0000
			154.9015	0.0000	9.9819	158.1199
			2455.5099	0.0000	158.1199	2506.1516
	N	834	811283.2369	0.0000	24506.7876	815125.8739
			0.0000	0.0000	0.0000	0.0000
			24506.7876	0.0000	740.3496	24623.0760
			815125.8739	0.0000	24623.0760	818987.4358
	W	285	2.1165	-0.1391	-0.1428	-0.0724
			-0.1391	0.0120	-0.0230	0.0016
			-0.1428	-0.0230	0.7046	0.1283
			-0.0724	0.0016	0.1283	0.0297

The density change equation is given in the following table.

Table 18. The fits for the prediction equation (13) for density change are given by forest unit.

FU	Origin	N	h_1	h_2	h_3	R^2	Average stem change (stems/ha)	Mean resid	Mean squared residual
PJ1	N	644	-0.0113*	0.00016*	-0.00043*	0.47	0.16551	-0.000292337	0.000257709
	P	156	-0.0128*	0.00025	-0.00084	0.26	0.79739	-0.000582551	0.001696809
PJ2	N	199	-0.0017	0.00004	-0.00056*	0.53	0.04367	0.001271501	0.000278114
	P	3	-0.0089*	0.00000	-0.00079	0.85	0.50449	-3.4694E-18	0.000126418
PO1	N	418	-0.0139*	0.00015	-0.00049*	0.27	0.19996	0.003518716	0.000570171
	P	1							
SB1	N	678	-0.0017	0.00001	-0.00071*	0.45	0.03828	0.002723993	0.000689467
SF1	N	26	-0.0020	0.00000	-0.00040	0.35	0.14025	-0.001662757	0.000237539
SP1	B	154	-0.0231*	0.00116*	-0.00137*	0.32	0.87378	0.000094030	0.000449165
	N	372	-0.0080*	0.00009*	-0.00059*	0.59	0.11311	-0.000283773	0.000379828
	W	277	-0.0063*	0.00035*	-0.00085*	0.65	0.89777	-0.000787406	0.000117665

8.4 Confidence intervals

Confidence intervals are given for selected management intensities (Figure 19 to Figure 22). Confidence intervals were calculated for basal area and gross total volume using the procedure in section 4.7. The results are presented here for gross total volume. The confidence intervals for gross total volume include the errors associated with estimating basal area and estimating volume from basal area and top height. The top height equations were taken from the literature.

In summary, the confidence intervals include errors arising from estimating basal area (from site index, age, and stocking) and from estimating gross total volume (from basal area and top height). The confidence intervals do not include errors associated with predicting top height, site index, stocking, age, and estimates of genetic gain. These errors can be considerable.

The assumption behind the confidence interval estimates is that the errors are independently and identically distributed as normal random variables with a mean of zero and a constant variance equal to the mean squared error. Some data are repeated measurements from the plots (not independent) so the mean squared error is likely underestimated. When the variance was obviously not constant, the observations were weighted in the regression routine to result in more constant variance. The confidence intervals are an approximation.

The confidence intervals for the growth predictions are given for 5 year predictions. Successive predictions will have considerably wider confidence intervals.

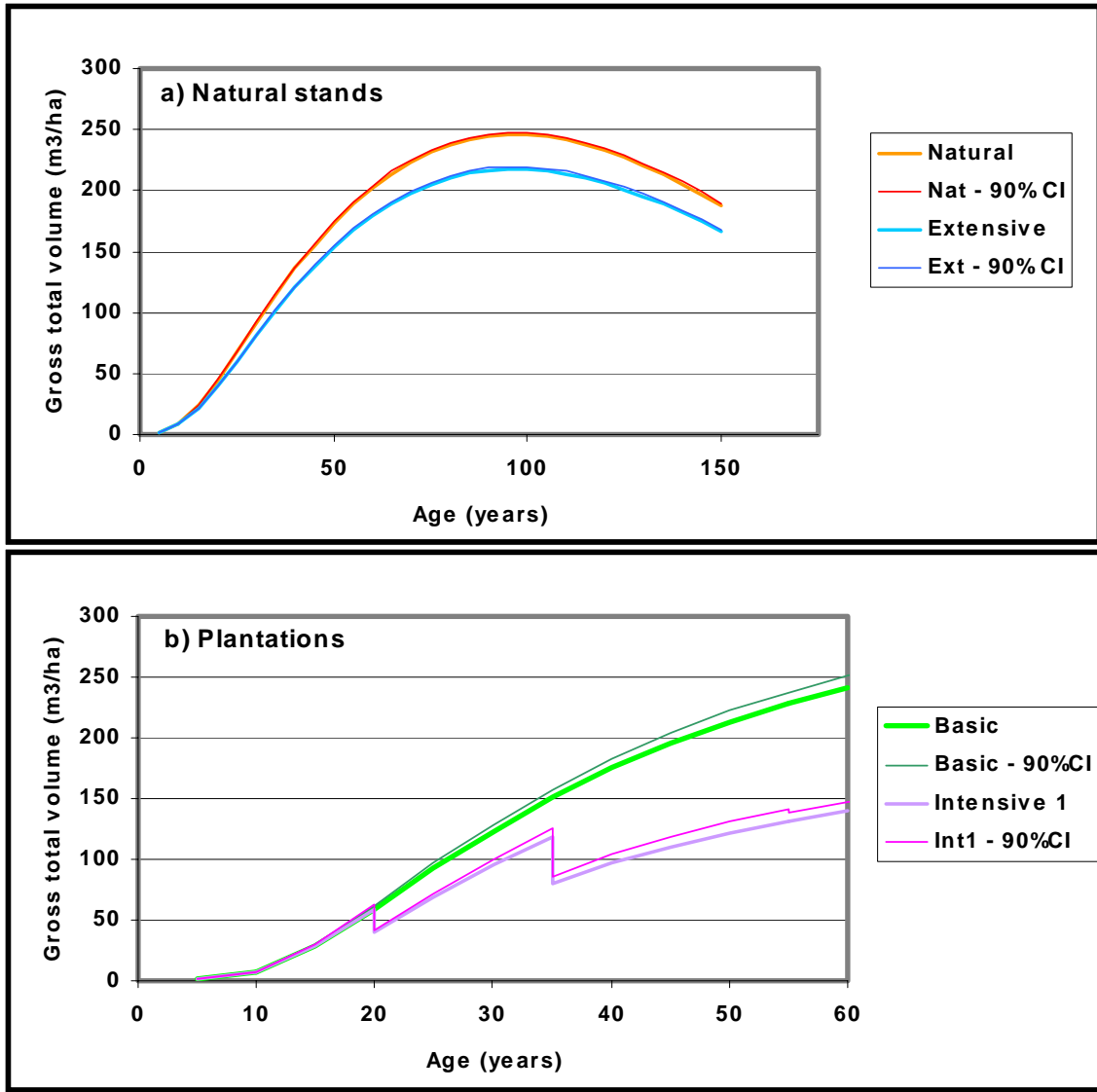


Figure 19. The 90% confidence intervals are given for yield predictions (natural stands) and for 5-year growth predictions (plantations) for the PJ1 forest unit. The management intensities associated with tree improvement are not given.

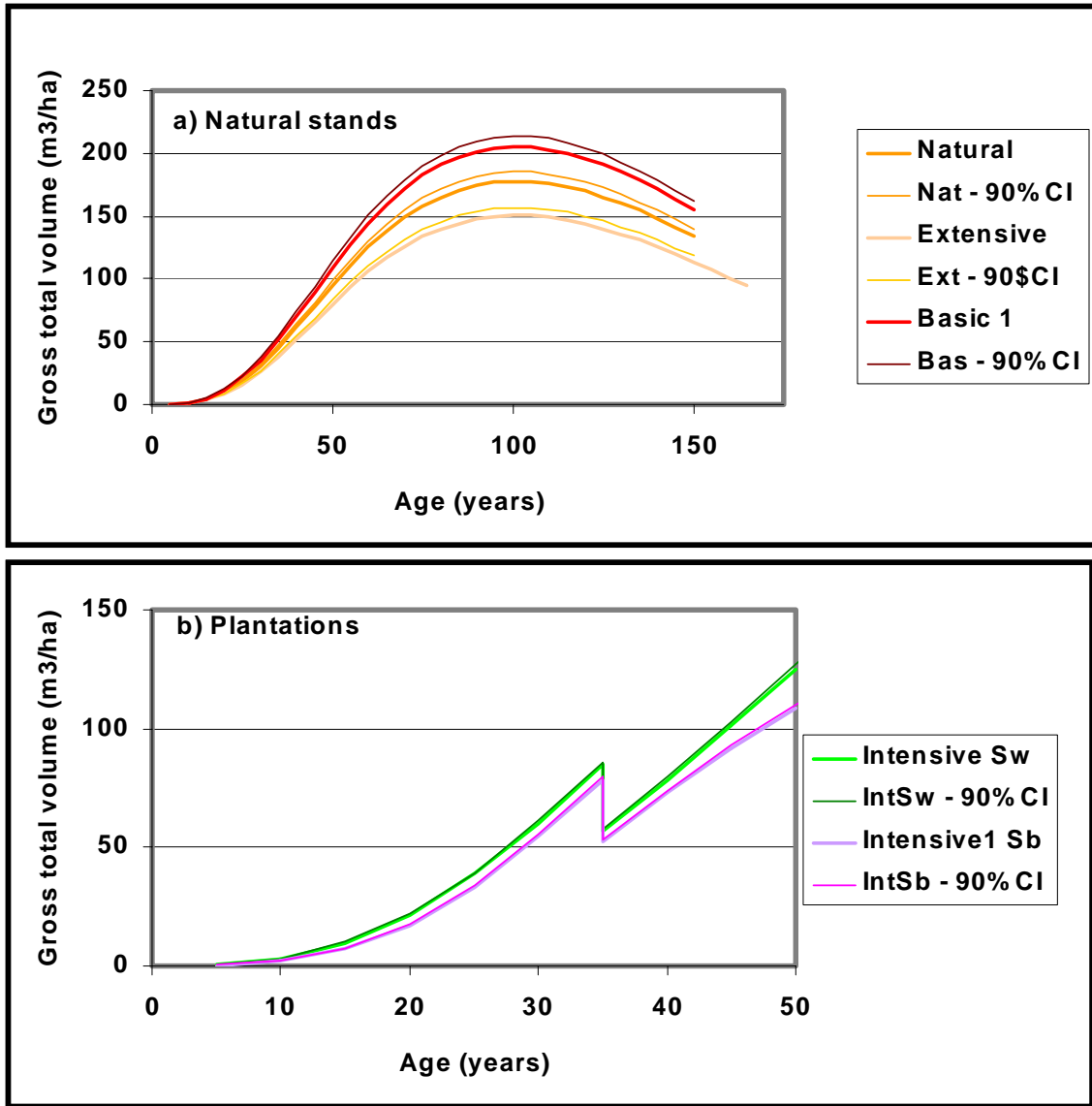


Figure 20. The 90% confidence intervals are given for yield predictions (natural stands) and for 5-year growth predictions (plantations) for the SP1 forest unit. The management intensities associated with tree improvement are not given.

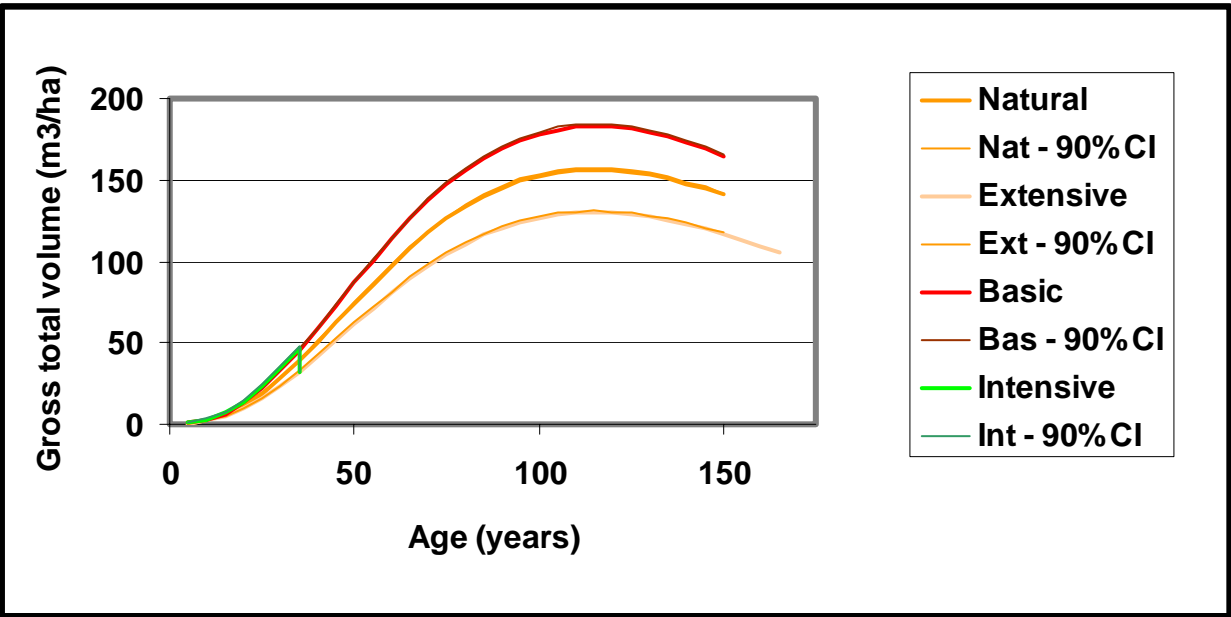


Figure 21. The 90% confidence intervals are given for yield predictions (natural stands) for the SB1 forest unit.

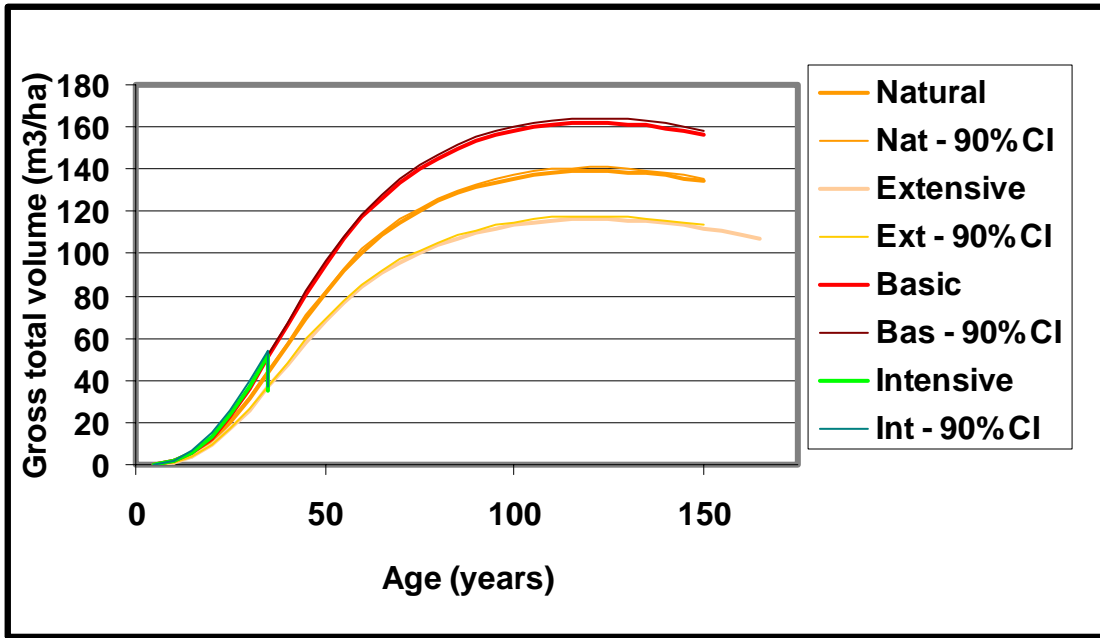


Figure 22. The 90% confidence intervals are given for yield predictions (natural stands) for the SF1 forest unit.

8.5 Residuals

The standard errors of the merchantable volume residuals are plotted in Figure 23. There is a tendency of the for the standard error to increase beyond age 150.

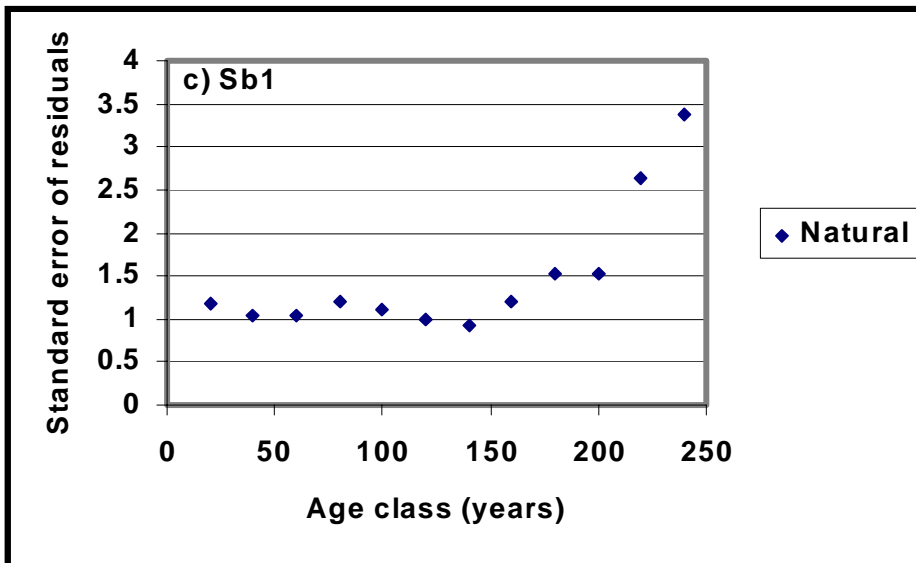
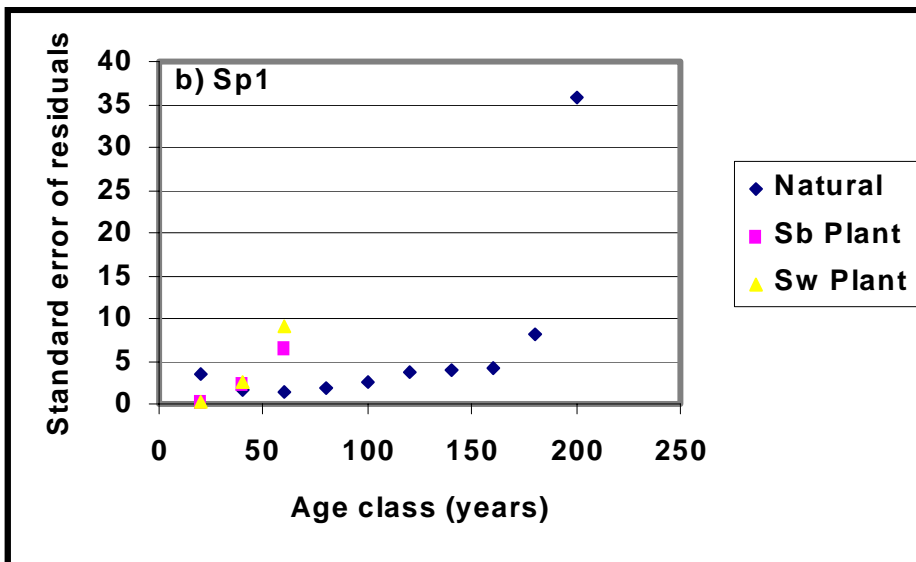
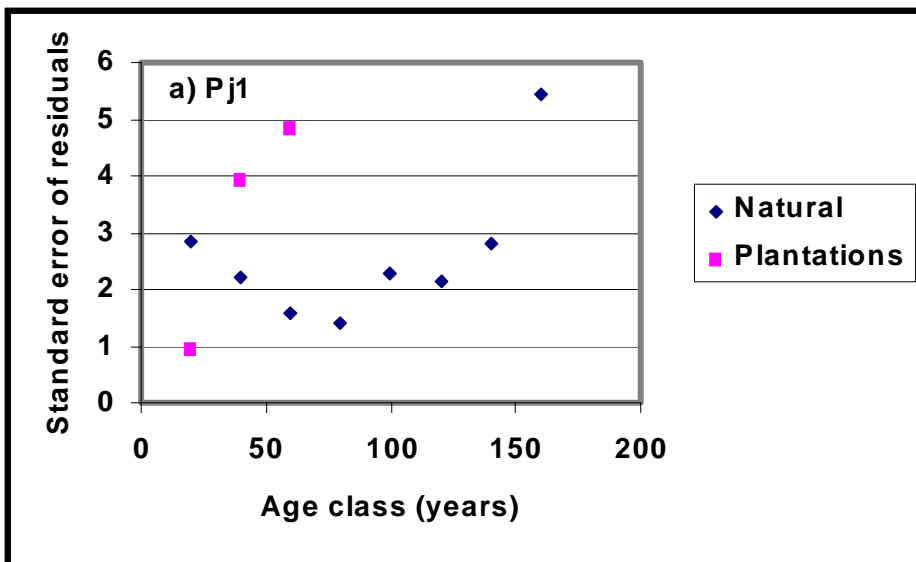


Figure 23. The standard error of the merchantable volume residuals are plotted by 20-year age class by forest unit and stand origin.

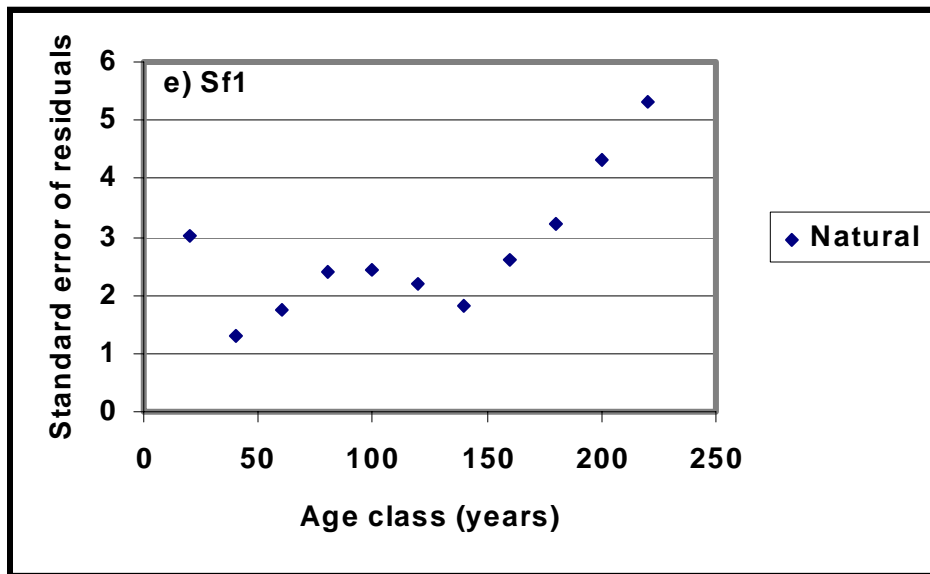
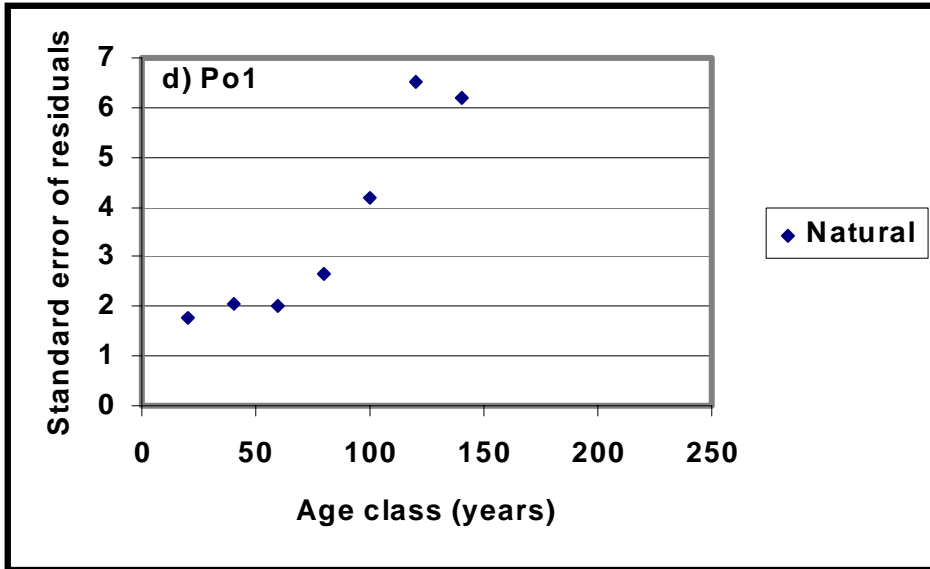


Figure 23. (cont.).